MANGALORE UNIVERSITY

M.Sc. Mathematics Choice Based Credit System (Semester Scheme) Programme from the academic year 2024-25

Preamble:

The syllabi for the M.Sc. Mathematics Programme with Choice Based Credit System (CBCS) were introduced in the academic year 2016-17. To align with global standards and provide hands-on experience, practical components were added, and the restructured syllabi were implemented in 2019-20. In 2022-23, further revisions were made by adding some soft-core courses, an additional open elective course, and replacing Scilab with Python programming for lab courses.

In the current syllabi, a hard core course "MTH 409 Linear Algebra" is introduced by merging the contents of the courses MTH 402: Linear Algebra-I and MTS 455: Linear Algebra-II. The soft core course MTS 507: Graph Theory, previously offered as an elective in the third semester, has been revised and is now offered as a compulsory soft core course, MTS 459: Graph Theory, in the second semester, replacing MTS 455: Linear Algebra-II. The syllabus of hard core courses MTH 453 Real Analysis-II, MTH 502 Complex Analysis-I, MTH 503 Measure and Integration, MTH 552 Complex Analysis-II, MTH 553 Functional Analysis, and the soft core courses MTS 513 Applied Algebraic Coding Theory, MTS 514 Operations Research, MTS 515 Design and Analysis of Algorithms, MTS 516 Advanced Number Theory, MTS 561 Cryptography, MTS 563 Advanced Graph Theory, and an open elective course MTE 512 Mathematical Finance are revised. The newly introduced soft core course "MTS 517: Classical Mechanics" will be available in the third semester, while the existing soft core course "MTS 509: Fluid Mechanics" has been revised and will now be offered as "MTS 564: Fluid Dynamics" in the fourth semester.

The course outcomes for all courses have been revised to meet NAAC requirements. The current syllabi incorporate recommendations from the U.G.C. Curriculum Development Committee and will be implemented starting the academic year 2024-25.

(*Revised as per the BOS meeting on 20.07.2024 to take the lead in the competitive/emulating industry/market based on the recent developments/inventions in the society.*)

Programme Outcome:

1. Provide a strong foundation in different areas of Mathematics, so that the students can compete with their contemporaries and excel in the various careers in Mathematics.
2. Develop abstract mathematical thinking.
3. Motivate and prepare the students to pursue higher studies and research, thus contributing to the ever increasing academic demands of the country.
4. Enrich the students with strong communication and interpersonal skills, broad knowledge and an understanding of multicultural and global perspectives, to work effectively in multidisciplinary teams, both as leaders and team members.

5. Facilitate integral development of the personality of the student to deal with ethical and professional issues, and also to develop ability for independent and lifelong learning.

Programme Specific Outcome:

1. Students will demonstrate in-depth knowledge of Mathematics, both in theory and application. They develop problem-solving skills and apply them independently to problems in pure and applied mathematics.
2. Students will attain the ability to identify, formulate and solve challenging problems in Mathematics. They assimilate complex mathematical ideas and arguments.
3. Students will be able to analyze complex problems in Mathematics and propose solutions using research based knowledge
4. Students will be able to work individually or as a team member or leader in uniform and multidisciplinary settings.
5. Students will develop confidence for self-education and ability for lifelong learning. Adjust themselves completely to the demands of the growing field of Mathematics by lifelong learning.
6. Effectively communicate about their field of expertise on their activities, with their peer and society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations.
7. Students will get the skills to answer competitive examinations such as JRF/NET, GATE, SET and other fellowships examinations conducted by premier institutions/agencies.

A. Consolidated List of Courses:

The following shall be the Courses of study in the four semesters M.Sc. Mathematics Programme (CBCS-PG) from the academic year 2024-2025.

Hard Core Courses:

First Semester	Second Semester
1. MTH 401 Algebra - I 2. MTH 409 Linear Algebra 3. MTH 403 Real Analysis - I	4. MTH 452 Algebra - II 5. MTH 453 Real Analysis - II 6. MTH 454 Topology

Third Semester	Fourth Semester
7. MTH 502 Complex Analysis - I 8. MTH 503 Measure and Integration 9. MTH 504 Multivariate Calculus and Geometry	10. MTP 551 Project Work 11. MTH 552 Complex Analysis – II 12. MTH 553 Functional Analysis

Soft Core Courses

First Semester	Second Semester
1. MTS 404 Numerical Analysis 2. MTS 407 Theory of Combinatorics 3. MTL 408 Practical -I	4. MTS 456 Ordinary Differential Equations 5. MTS 459 Graph Theory 6. MTL 458 Practical - II

Third Semester	Fourth Semester
7. MTS 505 Advanced Numerical Analysis 8. MTS 506 Commutative Algebra 9. MTS 508 Lattice Theory 10. MTS 513 Applied Algebraic Coding Theory 11. MTS 514 Operations Research 12. MTS 515 Design and Analysis of Algorithms 13. MTS 516 Advanced Number Theory 14. MTS 518 Classical Mechanics 15. MTL 517 Practical - III	16. MTS 554 Partial Differential Equations 17. MTS 555 Advanced Topology 18. MTS 557 Algebraic Number Theory 19. MTS 558 Calculus of Variations and Integral Equations 20. MTS 559 Mathematical Statistics 21. MTS 560 Computational Geometry 22. MTS 561 Cryptography 23. MTS 562 Finite Element Method with Applications 24. MTS 563 Advanced Graph Theory 25. MTS 564 Fluid Dynamics

Open Elective Courses

Second Semester	Third Semester
1. MTE 451 Discrete Mathematics and Applications.	2. MTE 501 Differential Equations and Applications 3. MTE 512 Mathematical Finance

Note:

1. All hard core courses are of 4 credits each and all are compulsory.
2. Practical courses are of 2 credits each and all are compulsory. For practical the student faculty ratio is 10:1. That is for every ten student one faculty to be allotted for effective implementation.
3. Soft core courses except practical courses are of 4 credits each. The soft core courses in the first two semesters are compulsory. In the third and fourth semesters student can choose any two soft core courses (other than practical courses) from the list of soft core courses offered in that semester.
4. Project work which is compulsory for every student, involves self study to be carried out by the student (on a research problem of current interest or on an advanced topic not covered in the syllabus) under the guidance of a supervisor.
5. Supervisor may be from the parent institution or from any other reputed institution/industry.
6. Project work shall be initiated in the third semester itself and the project report (dissertation) shall be submitted at the end of the fourth semester.
7. Project guidance shall be included in the teaching workload. Project guidance session for every two students will be considered as 1 hour per week. The student-to-faculty ratio for project supervision is 8:1.

B. Scheme of Instruction and Examination

First Semester

Course Code	Instruction Hours per week			Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
	Theory	PSS	Total					
MTH 401	4	2	6	4	3	70	30	100
MTH 402	4	2	6	4	3	70	30	100
MTH 403	4	2	6	4	3	70	30	100

MTS 404	4	2	6	4	3	70	30	100
MTS 407	4	2	6	4	3	70	30	100
MTL 408	4	-	4	2	3	35	15	50

Second Semester

Course Code	Instruction Hours per week			Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
	Theory	PSS	Total					
MTE 451	3	1	4	3	3	70	30	100
MTH 452	4	2	6	4	3	70	30	100
MTH 453	4	2	6	4	3	70	30	100
MTH 454	4	2	6	4	3	70	30	100
MTS 456	4	2	6	4	3	70	30	100
MTS 459	4	2	6	4	3	70	30	100
MTL 458	4	-	4	2	3	35	15	50

Third Semester

Course Code	Instruction Hours per week			Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
	Theory	PSS	Total					
MTE 501	3	1	4	3	3	70	30	100
MTE 512	3	1	4	3	3	70	30	100
MTH 502	4	2	6	4	3	70	30	100
MTH 503	4	2	6	4	3	70	30	100
MTH 504	4	2	6	4	3	70	30	100
MTS 505	4	2	6	4	3	70	30	100
MTS 506	4	2	6	4	3	70	30	100
MTS 507	4	2	6	4	3	70	30	100
MTS 508	4	2	6	4	3	70	30	100
MTS 509	4	2	6	4	3	70	30	100
MTS 513	4	2	6	4	3	70	30	100
MTS 514	4	2	6	4	3	70	30	100
MTS 515	4	2	6	4	3	70	30	100
MTS 516	4	2	6	4	3	70	30	100
MTS 518	4	2	6	4	3	70	30	100
MTL 517	4	-	4	2	3	35	15	50

Fourth Semester

Course Code	Instruction Hours per week			Credits	Duration of Examination in hours	University Examination Max. Marks	Internal Assessment Max. Marks	Total Marks
	Theory	PSS	Total					
MTP 551	6			4	-	70	30	100
MTH 552	4	2	6	4	3	70	30	100
MTH 553	4	2	6	4	3	70	30	100
MTS 554	4	2	6	4	3	70	30	100
MTS 555	4	2	6	4	3	70	30	100
MTS 557	4	2	6	4	3	70	30	100
MTS 558	4	2	6	4	3	70	30	100
MTS 559	4	2	6	4	3	70	30	100

MTS 560	4	2	6	4	3	70	30	100
MTS 561	4	2	6	4	3	70	30	100
MTS 562	4	2	6	4	3	70	30	100
MTS 563	4	2	6	4	3	70	30	100
MTS 564	4	2	6	4	3	70	30	100

PSS: There shall be 2 hours of problem solving sessions (PSS) per week for each course having 4 credits and 1 hour problem solving session for an open elective. PSS are to be considered as teaching hours and in the PSS students are required to solve the problems in presence of the course instructor.

Scheme of Evaluation for Internal Assessment Marks:

1. Theory Course:
   Each Theory Course shall carry 30 marks for internal assessment based on two tests of 90 minutes duration each.
2. Project Work:
   Project Work shall carry 30 marks for internal assessment based on two presentations by the student before a panel of faculty members of the department.
3. Practical:
   Each Practical shall carry 15 marks for internal assessment based on two tests of 90 minutes duration each.

Pattern of Semester Examination:

1. Theory Paper:
   Each question paper for the theory course shall contain EIGHT questions out of which FIVE are to be answered. All questions carry equal marks.
2. Project Report:
   The evaluation of a project report is by two examiners as per the regulations.
3. Practical Exam:
   Each Practical exam question paper shall contain TWO questions on lab programmes which are to be executed.

C. Syllabi of Each Semester

I Semester

MTH 401	Algebra- I	4 Credits (48 hours)

Course Objectives

1. **Develop a strong foundation in group theory:** Understand the fundamental concepts of groups, subgroups, cyclic groups, homomorphisms, isomorphisms, etc. Explore their applications in various mathematical contexts.
2. **Analyze symmetries and isometries:** Study the symmetries of plane figures and the properties of isometries to understand their mathematical significance.
3. **Explore advanced group theory concepts:** Delve into the class equation, p-groups, conjugation in symmetric groups, and the Sylow theorems. Learn their applications in different areas of mathematics.

4. **Understand the fundamentals of ring theory:** Grasp the basic definitions and properties of rings, polynomial rings, integral domains, fields, homomorphisms, ideals.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Understand and apply the laws of composition, recognize various types of groups (such as cyclic groups), and classify subgroups of given groups.
2. Demonstrate proficiency in understanding and using homomorphisms and isomorphisms, including the Correspondence Theorem to solve problems involving cosets and quotient groups.
3. Understand the concepts of isometries, finite groups of orthogonal operators, and apply these ideas to various mathematical problems.
4. Use group operations, Cayley's theorem, the class equation, and the Sylow theorems to analyze finite subgroups and conjugation in symmetric groups, and solve advanced problems in group theory.
5. Comprehend the definitions and properties of rings, polynomial rings, integral domains, fields, and their homomorphisms and ideals. Apply these concepts to solve problems involving quotient rings, adjoining elements, product rings, and maximal ideals.

Contents

Unit I – Groups (12 Hours)

Laws of Composition, Groups and Subgroups, Subgroups of the Additive Group of Integers, Cyclic groups, Homomorphisms, Isomorphisms, Equivalance Relation and Partitions, Cosets and Lagrange's Theorem, Modular Arithmetic, The Correspondence Theorem, Product Groups, Quotient Groups.

Unit II - Isometries and Operations on Groups (12 Hours)

Symmetry: Symmetry of plane figures, Isometries, Isometries of the plane, Finite groups of orthogonal operators on the plane.

Abstract Symmetry: Group Operations, The operation on Cosets, The counting Formula, Operations on subsets, Permutation Representations.

Unit III - Advanced Group Theory (12 Hours)

Finite subgroups of the Rotation Group, Cayley's theorem, The class equation, p-Groups, Conjugation in the symmetric group, Normalizers, The Sylow theorems and its Applications.

Unit IV – Ring Theory (12 Hours)

Definitions of rings, Polynomial Rings, integral domains, Fields and their basic properties, Homomorphisms and Ideals, Quotient Rings, Adjoining elements, Product rings, Fractions, Maximal Ideals.

References

[1] Michael Artin, *Algebra*, 2nd Ed., Prentice Hall of India, 2013.

[2] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.

[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

[4] V. A. Herimath Algebra - Set Theory, Natural Numbers and Group Theory, Narosa, 2022.

[5] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.

[6] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.

[7] Thomas W. Hungerford, *Algebra*, Springer, 2004.

[8] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.

[9] Serge Lang, *Algebra,* 3rd Ed., Springer, 2005.

MTH 402	Linear Algebra -I	4 Credits (48 hours)

Course Objectives

1. **Understand the fundamental concepts of vector spaces and linear transformations**: Grasp the principles of vector spaces, linear combinations, bases, dimension, linear transformations, null spaces, ranges, and matrix representations.
2. **Explore elementary matrices and determinants**: Learn about elementary matrix operations, matrix rank, inverses, and the properties and applications of determinants.
3. **Examine diagonalization and related matrix properties**: Study eigenvalues, eigenvectors, diagonalizability, The jordan form, Computation of invariant factors, and the Cayley-Hamilton theorem.
4. **Explore inner product spaces and associated linear operators**: Understand the Gram-Schmidt process, various linear operators and derive the spectral theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Understand and utilize vector space principles, linear transformations, matrix representations, null spaces, and ranges in solving mathematical problems.
2. Conduct elementary matrix operations, determine matrix rank and inverses, and calculate determinants of various orders with an understanding of their properties.
3. Find eigenvalues and eigenvectors, assess the diagonalizability of matrices, and apply related concepts such as the Cayley-Hamilton theorem.
4. Work with inner products, norms, the Gram-Schmidt orthogonalization process, orthonormal complements, and adjoint linear operators.
5. Understand and use normal and self-adjoint operators, unitary and orthogonal operators, orthogonal projections, the spectral theorem, singular value decomposition, pseudo-inverse, and the geometry of orthogonal operators in mathematical contexts.

Contents

Unit I (12 Hours)

Vector spaces: Recapitulation of Vector spaces, Linear combinations and system of linear equations, linear dependence and linear independence, Bases and dimension, maximal linearly independent subsets.

Linear transformations: Linear transformations, null spaces, and ranges, The matrix representation of a linear transformation, Composition of linear transformations and matrix multiplication, Invertibility and Isomorphisms, The change of coordinate matrix, Dual spaces.

Unit II (12 Hours)

Elementary matrices: Elementary matrix operations and elementary matrices, The rank of a matrix and matrix inverses, System of linear equations - theoretical and computational aspects.
Determinants: Determinants of order 2, Determinants of order *n*, Properties of determinants, A characterization of the determinant.

Unit III (12 Hours)

Diagonalization: Eigenvalues and eigenvectors, Diagonalizability, Invariant subspaces and Cyley-Hamilton theorem.
Cyclic subspaces and annihilators, Cyclic decomposition and rational form, The jordan form, Computation of invariant factors.

Unit IV (12 Hours)

Inner product spaces: Recapitulation of inner products and norms, The Gram-Schmidt orthogonalization process and Orthonormal complements.
The adjoint of a linear operator, Normal and self adjoint linear operators, Unitary and orthogonal operators and their matrices, Orthogonal projections and spectral theorem, Bilinear and Quadratic forms, The geometry of orthogonal operators.

References:

1. Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence – Linear Algebra, Prentice Hall of India, 4th Edition, 2014.
2. K. Hoffmann and R. Kunz – Linear Algebra, Prentice Hall of India, 2nd Edition, 2013.
3. S. Lang – Linear Algebra, Addison Wesley, London, 1970.
4. Michael Artin – Algebra, Prentice Hall of India, 2rd Edition, 2013.
5. S. Axler - Linear Algebra Done Right, Undergraduate Texts in Mathematics Springer, 4th Edition, 1997.
6. Larry Smith – Linear Algebra, Springer Verlag, 3rd Edition, 1998.
7. Gilbert Strang – Linear Algebra and its Applications, Cengage Learning, 4th Edition, 2006.
8. S. Kumaresan – Linear Algebra - A Geometric Approach, PHI, 2003.

MTH 403	Real Analysis-I	4 Credits (48 hours)

Course Objectives

1. **Understand properties of number systems**: Learn about real and complex number systems, ordered sets, fields, and Euclidean spaces.
2. **Learn basic topology**: Explore metric spaces, open and closed sets, compact sets, and connected sets.
3. **Study numerical sequences and series**: Investigate convergent sequences, series, root and ratio tests, and power series.
4. **Explore continuity and differentiation**: Examine limits, continuous functions, derivatives, mean value theorems, L'Hospital's rule, and Taylor's theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Utilize properties of real and complex numbers, ordered sets, fields, and Euclidean spaces to solve mathematical problems.
2. Define metric spaces, identify open and closed sets, compact sets, perfect sets, and connected sets, and apply the Heine-Borel theorem.

3. Determine convergence of sequences and series using Cauchy sequences, root and ratio tests, and summation by parts.
4. Analyze function limits, identify and categorize discontinuities, and apply concepts of continuity, compactness, and connectedness.
5. Compute derivatives, apply mean value theorems, use L'Hospital's rule, and apply Taylor's theorems to real and vector-valued functions.

Contents

Unit I - The Real and Complex Number System: (12 Hours)

Introduction, Ordered sets, Fields, The real field, The extended real number system, The complex field, Euclidean spaces, Inequalities. Finite, Countable and Uncountable sets, Countability of Rational Numbers.

Unit – II Basic Topology: (12 Hours)

Metric spaces- Definitions, Examples and Basic properties, Open and Closed sets, Compact sets, Hein-Borel Theorem, Perfect sets – Uncountability of real numbers, Connected sets – Discussion in real line under usual metric.

Unit III - Numerical Sequences and Series: (12 Hours)

Convergent sequences, Subsequences, Cauchy sequences, Upper and lower limits, Some special sequences, Series, Series of non-negative terms, The root and ratio tests, Power series, Summation by parts, Absolute convergence.

Unit IV – Continuity and Differentiation: (12 Hours)

Limits of functions, Continuous functions, Continuity and compactness, Continuity and connectedness, Discontinuities, Monotonic functions, Infinite limits and limits at infinity.

The derivative of a real function, Mean value theorems, The continuity of derivatives, L'Hospital's rule, Derivatives of higher order, Taylor's theorems, Differentiation of vector valued functions.

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[4] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[5] R. R. Goldberg, *Methods of Real Analysis,* 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[6] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[7] Russel A. Gordon, *Real Analysis - A First Course*, 2nd Ed., Pearson, 2011.

MTS 404	Numerical Analysis	4 Credits (48 hours)

Course Objectives

1. **Solving Transcendental and Polynomial Equations:** Understand methods for solving transcendental and polynomial equations, including convergence rates.
2. **Solving Linear Equations and Eigenvalue Problems**: Learn direct and iterative methods for solving systems of linear equations and eigenvalue problems.
3. **Interpolation and Approximation Techniques:** Study interpolation techniques and polynomial approximations, including least square approximations.
4. **Numerical Differentiation and Integration Methods:** Explore numerical differentiation and integration methods, including extrapolation and Newton-Cotes methods.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Use bisection, secant, Newton-Raphson, and other iteration methods, analyzing their convergence rates.
2. Apply direct methods (e.g., Gauss Elimination) and iterative methods (e.g., Jacobi, Gauss-Seidel), understanding convergence analysis and the power method.
3. Use Lagrange, Newton interpolations, finite differences, and Hermite interpolation for constructing interpolating polynomials and approximations.
4. Implement methods based on interpolation, finite differences, undetermined coefficients, and extrapolation for numerical differentiation.
5. Apply interpolation-based methods, Newton-Cotes formulas, and composite integration methods for accurate numerical integration.

Contents

Unit I - Transcendental and Polynomial Equations (12 Hours)

Introduction, The bisection method, Iteration methods based on first degree equation, Iteration methods based on second degree equation, Rate of convergence, Rate of convergence of Secant and Newton-Raphson method. Iteration methods - First order method, Second order method, Higher order methods. Polynomial equations, Descartes' Rule of Signs, The Birge-Vieta method, Ramanujan's method to find real /complex roots.

Unit II - System of Linear Equations and Eigen value problems (12 Hours)

Introduction, Direct Methods - Gauss Elimination Method, Gauss-Jordan Method, Triangularization Method, Cholesky Method. Iteration Methods - Jacobi Iteration method, Gauss-Seidel Iteration method, Convergence analysis, Eigen values and Eigen vectors. The Power Method.

Unit III - Interpolation and Approximation (12 Hours)

Introduction, Lagrange and Newton interpolations, Linear and Higher order interpolation, Finite difference operators, Interpolating polynomials using finite differences, Hermit interpolation, Approximations –Least Square Approximations.

Unit IV - Numerical Differentiation and Numerical Integration (12 Hours)

Numerical Differentiation: Introduction, Methods based on Interpolation, Methods based on finite differences, Methods based on undetermined coefficients, Extrapolation methods.

Numerical Integration: Methods based on Interpolation, Newton-Cotes methods, Composite Integration Methods.

References

[1] M. K. Jain, S. R. K. Iyengar, R. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Ed., New Age International, 2012.

[2] C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, Inc., 1999.

[3] A. Ralston and P. Rabinowitz, *A First Course in Numerical Analysis*, 2nd Ed., McGraw - Hill, New York, 1978.

[4] K. Atkinson, *Elementary Numerical Analysis*, 2nd Ed., John Wiley and Sons, Inc., 1994.

[5] P. Henrici, *Elements of Numerical Analysis*, John Wiley and Sons, Inc., New York, 1964.

MTS 407	**Theory of Combinatorics**	**4 Credits (48 hours)**

Course Objectives

1. **Understand Fundamentals of Logic and Proofs**: Learn propositional logic, predicates, quantifiers, and various proof methods and strategies.
2. **Apply Counting Techniques**: Master basic counting principles, permutations, combinations, and advanced techniques such as inclusion-exclusion and derangements.
3. **Utilize Generating Functions and Recurrence Relations**: Study generating functions, their calculation techniques, and solve recurrence relations using various methods.
4. **Apply Group Theory to Counting Problems**: Understand group actions, the Orbit-Stabilizer Theorem, and apply Polya's Counting Principle and Cycle Index Polynomial.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Use propositional logic, predicates, quantifiers, and various proof methods to solve problems and construct rigorous arguments.
2. Apply basic counting principles, permutations, combinations, and advanced methods like inclusion-exclusion and derangements to solve complex counting problems.
3. Compute and apply generating functions, including exponential generating functions, and use summation operators for problem-solving.
4. Solve first-order and second-order linear recurrence relations with constant coefficients, including non-homogeneous cases, using generating functions.
5. Utilize group actions, the Orbit-Stabilizer Theorem, Polya's Counting Principle, and Cycle Index Polynomial to solve counting and enumeration problems.

Contents

Unit I – The Fundamentals of Logic and Proofs (12 Hours)

Propositional Logic, Applications of Propositional Logic, Propositional Equivalence, Predicates and Quantifiers, Nested Quantifiers, Rules of Inferences, Introduction to Proofs, Proof Methods and Strategy.

Unit II – Counting Techniques (12 Hours)

Counting: The Basics of Counting, Pigeon-hole Principle, Permutations and Combinations, Binomial Coefficients and identities, Generalized Permutations and Combinations.

Advanced Counting Techniques: Principle of Inclusion-Exclusion, Generalizations of the Principle, Derangements, Rook Polynomials.

Unit III – Generating Functions and Recurrence Relations (12 Hours)

Generating Functions: Introductory Example, Calculation Techniques, Partition of integers, Exponential Generating Function, The Summation operator.

Recurrence Relations: The First Order Linear Recurrence Relations, Second Order Linear Homogeneous Recurrence Relations with Constant Coefficients, Non- homogeneous Recurrence Relations, The method of Generating Functions.

Unit IV – Applications of Group Theory in Counting (12 Hours)

Group Action, Orbit Stabilizer Theorem and its applications to Polya's Counting Principle, The Cycle index Polynomial, Polya's Theorem Special Case and Applications, The Pattern Inventory, Polya's Theorem General Case and Polya's Inventory Problems.

References

[1] Kenneth H. Rosen, *Discrete Mathematics and its Applications*, 7th Ed., McGraw Hill, 2012.

[2] Ralph P. Grimaldi, *Discrete Combinatorial Mathematics*, 5th Ed., Pearson, 2006.

[3] D. I. A. Cohen, *Basic Techniques of Combinatorial Theory*, John Wiley and Sons, New York, 1978.

[4] Fred S. Roberts, Barry Tesman, *Applied Combinatorics*, 2nd Ed., CRC Press, 2009.

[5] G. E. Martin, *Counting: The Art of Enumerative Combinatorics*, UTM, Springer, 2001.

MTL 408	**Practical -I**	**2 Credits (2 hours lab /week)**

Mathematics Practicals using Python Programming Language

Course Objectives

1. **Implement Basic Programming Constructs**: Develop programs to handle arrays, calculate factorials, and generate Fibonacci numbers using loops and conditionals.

2. **Perform Number System Conversions**: Write programs to convert between binary, octal, and decimal number systems using user-defined functions.
3. **Search, Sort, and Solve Equations**: Implement search and sorting algorithms, and apply methods to find roots of algebraic and transcendental equations.
4. **Apply Interpolation Techniques**: Develop programs to perform Lagrange, Newton Gregory, and Hermite interpolation methods for function approximation.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Write Python programs to find the largest/smallest element in an array and calculate factorials and Fibonacci numbers.
2. Convert numbers between binary, octal, and decimal formats using Python functions.
3. Use Python to perform linear and binary searches, and sort arrays in ascending and descending order.
4. Apply methods like Newton-Raphson, Secant, and Birge-Vieta to find real roots of equations.
5. Implement Lagrange, Newton Gregory, and Hermite interpolation methods to approximate functions.

List of Programs

1) Program to accept an array of numbers and print the largest/smallest among them (using ‘if’ – statement, ‘elif’-statement and for loop).
2) Program to calculate factorial of a number and program to print Fibonacci numbers using ‘for loop’.
3) Program to convert binary/octal number to decimal number and decimal number to binary/octal number using user defined functions.
4) Program to search an element in the array using linear and binary search.
5) Program to arrange a set of given integers in an ascending/descending order and print them.
6) Program to find roots of a quadratic equation.
7) Program to find a real root of a Algebraic/Transcendental equation using Newton Raphson Method/Chebyshev Method.
8) Program to find a real root of an Algebraic/Transcendental equation using Secant Method/Regula-Falsi Method.
9) Program to find a real root of a polynomial equation using Birge-Vieta Method.
10) Program to illustrate Lagrange interpolation.
11) Program to illustrate Newton Gregory Forward/Backward Difference interpolation methods.
12) Program to find the value of a function by using Hermite interpolation method.

Note: The above list may be changed annually with the approval of the PG BOS in Mathematics.

II Semester

MTE 451	Discrete Mathematics and Applications	3 Credits (36 hours)

Prerequisite: Basic Mathematics up to XII/PU.

Course Objectives

1. Understand the fundamentals of number theory and their applications to cryptography.
2. Master various counting techniques and their applications in combinatorics.
3. Comprehend the properties and structures of relations, including partially ordered sets and Boolean algebras.
4. Develop problem-solving skills in number theory, combinatorics, and order relations.

Course Outcomes

1. Apply modular arithmetic and divisibility rules to solve complex problems in number theory.
2. Utilize counting techniques, such as permutations, combinations, and recurrence relations, in practical applications.
3. Analyze and interpret properties of relations, including extremal elements and lattice structures.
4. Implement cryptographic algorithms and understand their mathematical foundations.
5. Design and optimize circuits using Boolean algebra and finite Boolean algebras.

Contents

Unit I – Basics of Number Theory and Introduction to Cryptography:
Divisibility and Modular Arithmetic, Integer Representations and Algorithms, Primes and Greatest Common Divisors, Solving Congruences, Applications of Congruences, Introduction to Cryptography. (12 Hours)

Unit II - Counting Techniques:
The Basics of Counting, The Pigeon-hole Principle, Permutations and Combinations, Binomial Coefficients and Identities, Generalized Permutations and Combinations, Recurrence Relations, Applications of Recurrence Relations, Solving Linear Recurrence Relations, Generating Functions. Principle of Inclusion-Exclusion, Applications of Inclusion-Exclusion. (12 Hours)

Unit III - Order Relations and Structures:
Product Sets and Partitions, Relations, Properties of Relations, Partially Ordered Sets, Extremal Elements of Partially Ordered Sets, Lattices, Finite Boolean Algebras, Functions on Boolean Algebras, Circuit Designs. (12 Hours)

References

[1] Kenneth H. Rosen, *Discrete Mathematics and Its Applications*, 7th Ed., Tata Mc-Graw-Hill, 2012.

[2] Bernard Kolman, Robert C. Busby, Sharon Cutler Ross, *Discrete Mathematical Structures*, 3rd Ed., Prentice Hall, 1996.

[3] Grimaldi R, *Discrete and Combinatorial Mathematics*, 5th Ed., Pearson, Addison Wesley, 2004.

MTH 452	Algebra - II	4 Credits (48 hours)

Course Objectives

1. **Understand Factoring Techniques:** Study integer and polynomial factorization, including unique factorization domains, Euclidean domains, and irreducibility tests.
2. **Explore Field Extensions:** Learn about field extensions, including the degree of extension, algebraic vs. transcendental elements, and irreducible polynomials.
3. **Examine Advanced Field Concepts:** Understand field isomorphisms, splitting fields, primitive elements, algebraically closed fields, and finite fields.
4. **Apply Galois Theory:** Explore automorphisms, fixed fields, Galois extensions, and the main theorem of Galois theory, including its applications to polynomial equations.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Apply techniques for factoring integers and polynomials, including unique factorization and irreducibility tests.
2. Understand and find the degree of field extensions, and differentiate between algebraic and transcendental elements.
3. Determine isomorphisms between field extensions, and understand concepts like splitting fields and primitive elements.
4. Use Galois theory to analyze automorphisms, fixed fields, and solve cubic, quadratic, and quintic equations.
5. Explore the properties of finite fields and apply field extension concepts to solve related problems.

Contents

Unit I - Factoring:

Factoring Integers, Unique Factorization Domains, Euclidean domains, Content of polynomials, Primitive polynomials, Gauss lemma, Unique factorization in R[x], where R is a UFD, Factoring Integer Polynomials, Irreducibility test mod *p*, Eisenstein's criterion, Gauss primes.

(12 Hours)

Unit II – Fundamentals of Field Extensions

Definition and Examples, Characteristic of a Field, The Degree of Field Extension, Algebraic and Transcendental Elements, Finding the irreducible Polynomial, Ruler and compass constructions.

(12 Hours)

Unit III –Field Extensions and Finite Fields

Isomorphism of field extensions, Adjoining roots, Splitting fields, Primitive elements, Algebraically closed fields, The fundamental theorem of algebra, Finite fields and their properties.

(12 Hours)

Unit IV - Galois Theory:

Automorphisms and Fixed Fields, Galois Extensions, The Main Theorem of Galois Theory, Illustrations of the Main theorem, Cubic Equations, Quadratic Equations, Roots of Unity, Quintic Equations. (12 Hours)

References

[1] Michael Artin, *Algebra,* 2nd Ed., Prentice Hall of India, 2013.

[2] J. B. Fraleigh, *A First Course in Abstract Algebra*, 7th Ed., Addison Wesley, 2003.

[3] I. N. Herstein, *Topics in Algebra*, 2nd Ed., John Wiley & Sons, 2006.

[4] Joseph A. Gallian, *Contemporary Abstract Algebra*, 8th Ed., Cengage Learning India, 2013.

[5] Paul B. Garrett, *Abstract Algebra*, CRC press, 2007.

[6] Thomas W. Hungerford, *Algebra*, Springer, 2004.

[7] David S. Dummit and Richard M. Foote, *Abstract Algebra*, 3rd Ed., Wiley, 2004.

[8] Serge Lang, *Algebra,* 3rd Ed., Springer, 2005.

MTH 453	Real Analysis - II	4 Credits (48 hours)

Course Objectives

1. **Understand Riemann-Stieltjes Integral**: Learn definitions, existence, properties, and applications of the Riemann-Stieltjes integral, including integration of vector-valued functions and rectifiable curves.
2. **Explore Sequences and Series of Functions**: Analyze point-wise and uniform convergence, Cauchy criterion for uniform convergence, and their implications for continuity, integration, and differentiation.
3. **Study Functions of Several Variables**: Master differentiation techniques, partial derivatives, directional derivatives, and key theorems such as the Contraction Principle, Inverse Function Theorem, and Implicit Function Theorem.
4. **Analyze Special functions and Function Spaces**: Understand special functions like exponential, trigonometric functions, gamma functions and their properties, and explore spaces of continuous functions, equicontinuous families, and approximation theorems.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Compute and analyze Riemann-Stieltjes integrals, understanding their properties and applications to vector-valued functions and rectifiable curves.
2. Assess point-wise and uniform convergence of function sequences, and understand their effects on continuity, integration, and differentiation.
3. Use partial derivatives, directional derivatives, and apply the Inverse Function and Implicit Function Theorems to functions of multiple variables.
4. Analyze and work with special functions, including exponential, logarithmic, and trigonometric functions.

5. Work with spaces of continuous functions, equicontinuous families, and apply Weierstrass polynomial approximation to solve related problems.

Contents

Unit I - The Riemann-Stieltjes Integral:
Definition and existence of integrals, Properties of integral, Integration and differentiation, Integration of vector-valued functions, Rectifiable curves. (12 Hours)

Unit II - Sequences and Series of Functions:

Discussion of main problem, Point-wise Convergence and Uniform convergence, Cauchy criterion for Uniform Convergence, Uniform convergence and continuity, Uniform convergence and integration, Uniform convergence and differentiation. (12 Hours)

Unit III – Special Functions and Weierstrass Theorem
The spaces $C(X)$ and $B(X)$, Equicontinuous families of functions, Weierstrass Polynomial approximation theorem and its applications.
Some Special Functions: Power series, The exponential and logarithmic functions, The trigonometric functions, The Gamma function. (12 Hours)

Unit IV - Functions of Several Variables:
Differentiation, Partial Derivatives, Directional derivatives, The Contraction Principle, The Inverse Function Theorem, The Implicit Function Theorem. (12 Hours)

References

[1] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, 1976.

[2] Robert. G. Bartle, *The Elements of Real Analysis*, 2nd Ed., Wiley International Ed., New York, 1976.

[3] Ajith Kumar and S. Kumaresan, *A Basic Course in Real Analysis*, CRC Press, 2014.

[4] Serge Lang, *Analysis* I, Addison Wesley Publishing Company, 1968.

[5] T. M. Apostol, *Mathematical Analysis*, 2nd Ed., Narosa Publishers, 1985.

[6] R. R. Goldberg , *Methods of Real Analysis*, 2nd Ed., Oxford & I. B. H. Publishing Co., New Delhi, 1970.

[7] N. L. Carothers, *Real Analysis*, Cambridge University Press, 2000.

[8] Russel A. Gordon, *Real Analysis - A First Course*, 2nd Ed., Pearson, 2011.

MTH 454	Topology	4 Credits (48 hours)

Course Objectives

1. **Understand Topological Spaces**: Learn the definition of topological spaces, including examples, elementary concepts, and open bases and subbases.
2. **Explore Compactness**: Study compact spaces, product spaces, and Tychonoff's theorem in the context of topology.
3. **Analyze Separation Axioms**: Investigate T1 and Hausdorff spaces, completely regular and normal spaces, and understand Urysohn's lemma, Tietze extension theorem, and Urysohn imbedding theorem.
4. **Examine Connectedness**: Learn about connected spaces, components, totally disconnected spaces, and locally connected spaces.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Identify and work with different types of topological spaces, open bases, and function algebras.
2. Demonstrate understanding of compact spaces and apply Tychonoff's theorem to problems involving product spaces.
3. Distinguish between various separation properties, and apply Urysohn's lemma, Tietze extension theorem, and Urysohn imbedding theorem.
4. Identify and analyze connected, totally disconnected, and locally connected spaces, and understand their components.
5. Use fundamental theorems and concepts to solve problems related to compactness, separation, and connectedness in topological spaces.

Contents

Unit I - Topological Spaces:
The definition and some examples, Open sets, Elementary concepts - Closed sets, Closure of a set, Kuratowski's Closure Axioms, Open bases and open subbases, Lindelof Theorem, Weak topologies, (12 Hours)

Unit II – Function algebras and Compact Spaces:
The function algebras *C(X, R)* and *C(X, C).* Compact Spaces, The Heine-Borel Theorem, Product spaces, Tychonoff 's theorem, The Generalized Heine-Borel Theorem, Compactness for metric spaces. (12 Hours)

Unit III - Separation:
T_1-Spaces and Hausdorff spaces, Completely regular spaces and Normal spaces, Urysohn's lemma and Tietze extension theorem, The Urysohn imbedding theorem. (12 Hours)

Unit IV - Connectedness:

Connected spaces, The components of a space, Totally disconnected spaces, Locally connected spaces. (12 Hours)

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

[2] J. R. Munkres, *Topology*, 2nd Ed., Pearson Education, Inc, 2000. (Add first Chapter).

[3] S. Willard, *General Topology*, Addison Wesley, New York, 1968.
[4] J. Dugundji, *Topology*, Allyn and Bacon, Boston, 1966.
[5] J. L. Kelley, *General Topology*, Van Nostrand Reinhold Co., New York, 1955.

MTS 456	Ordinary Differential Equations	4 Credits (48 hours)

Course Objectives

1. **Study Linear Differential Equations:** Understand linear dependence, the Wronskian, and methods for solving linear differential equations, including variations of parameters and equations with constant coefficients.
2. **Explore Power Series Solutions:** Analyze second-order linear equations with ordinary and singular points, including Legendre and Bessel equations.
3. **Examine Systems of Linear Differential Equations:** Learn to solve systems of first-order equations, apply the fundamental matrix, and address non-homogeneous systems and systems with periodic coefficients.
4. **Understand Existence and Uniqueness:** Investigate the existence and uniqueness of solutions for differential equations, including methods of approximation and Picard's theorem.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Apply methods like the Wronskian, variation of parameters, and handle equations with constant coefficients.
2. Solve second-order linear equations using power series, and work with Legendre and Bessel equations.
3. Solve first-order linear systems, apply the fundamental matrix, and manage systems with periodic coefficients.
4. Use Picard's theorem and methods of successive approximation to analyze solutions and address non-uniqueness.
5. Apply techniques to determine the existence, uniqueness, and continuation of solutions to differential equations.

Contents

Unit I - Linear Differential Equations of Higher Order:
Linear dependence and the Wronskian, Basic theory for linear equations, Method of variation of parameters, Reduction of n^{th} order linear homogeneous equation, Homogeneous and non-homogeneous equations with constant coefficients.
(12 Hours)

Unit II - Solutions in Power Series:
Second order linear equations with ordinary points, Legendre equation and Legendre polynomials, Second order equations with regular singular points, Bessel equation. (18 Hours)

Unit III - Systems of Linear Differential Equations:

Systems of first order equations, Existence and uniqueness theorem. The fundamental matrix, Non-homogeneous linear systems, Linear systems with periodic coefficients. (12 Hours)

Unit IV - Existence and Uniqueness of solutions :
Equations of the form $x' = f(t,x)$, Method of successive approximation, Lipschitz condition, Picard's theorem, Non uniqueness of solutions, Continuation of solutions. (6 Hours)

References

[1] S. G. Deo and V. Raghavendra, *Ordinary Differential Equations and Stability Theory*, Tata McGraw Hill, 1980.

[2] A. Coddington, *An Introduction to Ordinary Differential Equations*, Prentice Hall of India, 2013.

[3] A. Coddington and N. Levinson, *Theory of Ordinary Differential Equations*, Krieger, 1984.

[4] M. W. Hirsh and S. Smale, *Differential Equations, Dynamical Systems and Linear Algebra*, Academic Press, New York, 1974. 5. V. I. Arnold, Ordinary Differential Equations, MIT Press, Cambridge, 1981.

[5] Shepley L. Ross, *Differential Equations*, Wiley, 2004.

MTS 459	Graph Theory	4 Credits (48 hours)

Course Objectives

1. **Understand Fundamental Graph Theory Concepts**: Gain a thorough understanding of the basic concepts in graph theory, including subgraphs, vertex degrees, paths, connectedness, and various operations and products on graphs.
2. **Explore Connectivity in Graphs**: Develop knowledge of connectivity in graphs, including vertex and edge cuts, blocks, and important theorems such as Menger's theorem.
3. **Analyze Trees and Special Graphs**: Study the properties and characterization of trees, understand the concepts of centers and centroids, and learn methods for counting spanning trees. Explore the properties of Eulerian and Hamiltonian graphs.
4. **Learn Graph Colorability**: Understand the principles of graph colorability, including the chromatic number, and theorems related to coloring such as the Five Color Theorem and the chromatic polynomial.

Course Outcomes

1. Ability to define and work with subgraphs, calculate degrees of vertices, identify paths and connected components, and apply various operations and products on graphs.
2. Proficiency in analyzing graph connectivity, including identifying and utilizing vertex and edge cuts, understanding the concept of blocks, and applying Menger's theorem to solve problems.
3. Understanding the definition and characterization of trees, determining centers and centroids, counting spanning trees, and analyzing the properties of Eulerian and Hamiltonian graphs.

4. Ability to determine the chromatic number of graphs, apply the Five Color Theorem, and compute the chromatic polynomial, as well as understanding the implications of graph colorability in various contexts.
5. Enhanced problem-solving skills in graph theory, enabling the application of theoretical concepts to practical problems and the development of strategies for addressing complex graph-related challenges.

Contents

Unit I - **Basic Properties of Graphs**
Introduction, Basic concepts, subgraphs, Degrees of Vertices, Paths and connectedness, The problem of Ramsey, Extremal graphs, Intersection graphs, Operations on graphs, Graph products.
(12 Hours)

Unit II
Connectivity, Vertex cuts and Edge cuts, Connectivity and Edge connectivity, Blocks, Menger;s theorem,.
(12 Hours)

Unit III
Trees - Definition, Characterization, and Simple properties, Centers and centroids, Counting the number of Spanning trees.
Eulerian and Hamiltonian graphs - Eulerian graphs, Hamiltonian graphs and its properties,
(12 Hours)

Unit IV:
Colorability: The chromatic number, The Five Color Theorem, The chromatic polynomial.
Planar Graphs: Planar and non-planar graphs, Euler formula and its consequences, Dual of a plane graph. (12 Hours)

References:

1. R.Balakrishnan and K.Ranganathan – A textbook of Graph Theory, Springer-Verlag, 2000.
2. F. Harary – Graph Theory, Addison-Wesley Series in Mathematics, 1969.
3. Narsingh Deo – Graph Theory with Applications to Engineering and Computer Science, Prentice Hall of India, 1987.
4. Bela Bollabas – Modern Graph theory, Springer, 1998.
5. Douglass B. West – Introduction to Graph Theory, Prentice Hall of India, New Delhi, 1996.
6. O. Ore – Theory of Graphs, American Mathematical Society, Providence, Rhode Island, 1967.

MTL 457	Practical - II	2 Credits (2 hours lab /week)

Mathematics Practicals using Python Programming Language

Course Objectives

1. **Visualize Mathematical Functions**: Develop programs to plot labeled graphs of elementary functions, plane curves, space curves, and surfaces using Python.
2. **Manipulate Matrices**: Write Python programs to compute matrix operations such as transpose, trace, determinant, norm, and perform matrix addition, subtraction, multiplication, and inversion.

3. **Solve Linear Systems**: Implement methods to check the consistency of linear systems, and solve systems using matrix inversion, Cramer's rule, Gauss Elimination, and Gauss-Jordan methods.
4. **Apply Numerical Methods**: Use iterative methods like Jacobi and Gauss-Seidel, and numerical techniques like the Power Method to find eigenvalues and eigenvectors of matrices.

Course Outcomes

Upon successful completion of this course, students will be able to:

1. Create clear, labeled graphs of elementary functions, plane curves, space curves, and surfaces using Python libraries.
2. Calculate matrix transpose, trace, determinant, norm, and perform matrix arithmetic and inversion using Python.
3. Check the consistency of linear systems and find solutions using methods like matrix inversion, Cramer's rule, Gauss Elimination, and Gauss-Jordan.
4. Solve systems of equations using Jacobi and Gauss-Seidel iterative methods.
5. Use the Power Method to numerically determine the largest/smallest eigenvalue and corresponding eigenvector of a matrix.

List of Programs

1) Program to plot a neat labeled graph of elementary functions on the same plane.
2) Program to obtain the graph of plane curves - cycloid and astroid in separate figure on a single run.
3) Program to obtain a neat labeled graph of space curves - elliptical helix and circular helix in separate figure on a single run.
4) Program to obtain a neat labeled graph of surfaces - elliptic paraboloid and hyperbolic paraboloid in separate figure on a single run.
5) Program to find the Transpose, Trace, Determinant and Norm of a matrix.
6) Program to find sum, difference and product and inverse (if exists) of matrices.
7) Program to check whether the given system of linear equations are consistent.
8) Program to find solution to a system of linear equations by matrix inversion method (check for all conditions on input matrix).
9) Program to find solution to a system of linear equations by Cramer's rule (check for all conditions on input matrix).
10) Program to solve a system of equations using Gauss Elimination Method and Gauss Jordan Method.
11) Program to find the solution of a system of equations using Jacobi Iterative Method/Gauss Seidal Method.
12) Program to find the numerically largest/smallest eigenvalue and corresponding eigenvector of a matrix by using Power Method.

Note: The above list may be changed annually with the approval of the PG BOS in Mathematics.

III Semester

MTE 501	Differential Equations and Applications	3 Credits (36 hours)

Prerequisite: Basic Mathematics up to XII/PU.

Course Objectives

1. **Understand Solution Methods**: Recapitulate methods for solving first-order differential equations and their applications.
2. **Analyze Applications**: Explore applications of first and second-order ordinary differential equations in physics, chemistry, and engineering.
3. **Solve Complex Problems**: Develop skills to solve second-order linear differential equations using power series solutions.
4. **Explore Special Functions**: Gain knowledge of special functions in mathematical physics, including Bessel functions and various polynomials.

Course Outcomes

1. Apply methods to solve first-order ordinary differential equations and relate them to real-world problems such as dynamics and chemical reactions.
2. Solve second-order ordinary differential equations and apply them to problems involving vibrations, electric circuits, and more using Laplace transforms.
3. Use power series methods to solve second-order linear differential equations and understand their mathematical properties.
4. Gain proficiency in working with special functions like Bessel functions, Legendre polynomials, and others, essential in mathematical physics.
5. Implement learned techniques to tackle practical problems in physics and engineering, demonstrating a deep understanding of differential equations and their applications.

Contents

Unit I

Recapitulation of methods of solutions of first order differential equations, Applications of First Order Ordinary Differential Equations - Simple problems of dynamics - falling bodies and other motion problems, Simple problems of Chemical reactions and mixing, Simple problems of growth and decay. (10 Hours)

Unit II

Applications of Second Order Ordinary Differential Equations - Undamped simple harmonic motion, damped vibrations, Forced vibrations, Problems on simple electric circuits – Laplace transforms. (10 Hours)

Unit III

Power series solutions of Second Order Linear Differential Equations, their mathematical properties. Special Functions of Mathematical Physics - Bessel functions, Legendre polynomials, Chebyshev polynomials, Hermite polynomials and Laguerre polynomials.

(16 Hours)

References

[1] G. F. Simmons, *Differential Equations with Applications and Historical Notes*, Tata McGraw-Hill, New Delhi, 1991.

[2] D. Rainville and P. Bedient, *Elementary course on Ordinary Differential Equations*, Macmillan, New York, 1972.

[3] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol. I, Tata McGraw Hill, New Delhi, 1975.

MTE 512	MATHEMATICAL FINANCE	3 Credits (36 hours)

Prerequisite: Basic Mathematics up to X standards.

Course Objectives

1. **Understand Financial Mathematics**: Learn the fundamental concepts and calculations related to time value of money, interest rates, and annuities.
2. **Apply Portfolio Theory**: Develop skills in portfolio construction, risk management, and optimization.
3. **Explore Financial Markets**: Gain knowledge about different financial markets, instruments, and trading mechanisms.
4. **Analyze Financial Instruments**: Understand the basics of bonds, equities, and derivatives, and their roles in financial markets.

Course Outcomes

1. **Perform Financial Calculations**: Accurately compute present and future values, interest rates, and loan amortization schedules.
2. **Construct Efficient Portfolios**: Apply mean-variance analysis and CAPM to create optimized investment portfolios.
3. **Measure and Manage Risk**: Utilize various risk measures and techniques to manage financial risks effectively.
4. **Navigate Financial Markets**: Demonstrate a comprehensive understanding of the structure and functioning of different financial markets.
5. **Evaluate Financial Instruments**: Analyze and value various financial instruments, including bonds, stocks, and derivatives.

Contents

Unit I: Fundamentals of Financial Mathematics (12 Hours)

Introduction to Financial Mathematics: Basic concepts, time value of money, present and future value. Simple and Compound Interest: Calculations, differences, applications in finance.

Annuities and Perpetuities: Definitions, formulas, applications.

Loan Amortization: Methods, schedules, examples.

Bonds and Securities: Pricing, yield, duration, and convexity.

Basic Probability and Statistics: Probability distributions, expected value, variance, applications in finance.

Unit II: Portfolio Theory and Risk Management **(12 Hours)**

Introduction to Portfolio Theory: Diversification, risk, return. Mean-Variance Analysis: Efficient frontier, capital market line, security market line. Capital Asset Pricing Model (CAPM): Assumptions, derivation, applications. Risk Measures: Value at risk (VaR), conditional VaR, applications. Portfolio Optimization: Techniques, constraints, applications. Introduction to Derivatives: Forwards, futures, options, and swaps, basic pricing and applications.

Unit III: Financial Markets and Instruments **(12 Hours)**

Overview of Financial Markets: Types, functions, role in the economy. Equity Markets: Stock exchanges, trading mechanisms, stock indices. Fixed Income Markets: Treasury, corporate, and municipal bonds. Derivatives Markets: Structure, participants, trading strategies. Market Efficiency and Behavioral Finance: Efficient market hypothesis, behavioral finance anomalies. Financial Instruments and Innovations: Securitization, structured products, financial engineering.

References:

[1] Steven Roman, *Introduction to the Mathematics of Finance*, Springer; 2004th edition.

[2] David G. Luenberger, *Investment Science*, Oxford University Press, 2nd edition 2014.

[3] Samuel A. Broverman, *Mathematics of Investment and Credit*, 4th ed., ACTEX Publications, 2008.

[4] Stephen G. Kellison, *The Theory of Interest*, 3rd ed., McGraw-Hill, 2009.

[5] John McCutcheon and William F. Scott, *An Introduction to the Mathematics of Finance*, Elsevier Butterworth-Heinemann, 1986.

[6] Petr Zima and Robert L. Brown, *Mathematics of Finance,* 2nd ed., Schaum's Outline Series, McGraw-Hill, 1996.

MTH 502	**Complex Analysis - I**	**4 Credits (48 hours)**

Course Objectives

1. **Understand Complex Numbers**: Develop a solid foundation in complex numbers, including geometric interpretation, square roots, and rational powers.
2. **Explore Complex Functions**: Learn the topology of the complex plane, one-to-one and onto functions, limits, and continuity.
3. **Analyze Differentiability**: Study differentiability, Cauchy-Riemann equations, harmonic functions, and power series.
4. **Investigate Complex Integrals**: Examine complex line integrals, the Cauchy-Goursat theorem, and the winding number.
5. **Apply Integral Theorems**: Apply Cauchy's integral formula, Morera's theorem, and compute line integrals.

Course Outcomes

1. Demonstrate proficiency in complex number operations, geometric interpretations, rational powers and stereographic projection.

2. Apply topological concepts of the complex plane to understand the continuity and differentiability of complex functions.
3. Solve problems involving differentiability, Cauchy-Riemann equations, and harmonic functions.
4. Evaluate complex line integrals using the Cauchy-Goursat theorem and understand the concept of winding number.
5. Compute integrals using Cauchy's integral formula, understand Morera's theorem, and develop Taylor and Laurent series.

Contents

Unit I
Recapitulation of Complex numbers, Geometric interpretation, Square roots.
Rational powers of a complex number, Topology of the complex plane, One-to-one and onto functions, Concepts of limit and continuity, Stereographic projection, Sequences and series of functions.
(12 Hours)

Unit II
Differentiability and Cauchy-Riemann equations, Harmonic functions, Power series as an analytic function, Exponential and trigonometric functions, Logarithmic function, Inverse functions.
(12 Hours)

Unit III
Curves in the complex plane, Properties of complex line integrals, Cauchy-Goursat theorem, Consequence of simply connectivity, Winding number or index of a curve, Homotopy version of Cauchy's theorem. (12 Hours)

Unit IV
Cauchy integral formula and computation of line integrals, Morera's theorem, Existence of harmonic conjugate, Taylor's theorem, Zeros of analytic functions, Laurent series.
(12 Hours)

References:

1. S. Ponnusamy - Foundations of Complex Analysis, Narosa, New Delhi, 2nd edition, 2005.
2. S. Ponnuswamy and H. Silverman – Complex Variables with Applications, Birkhäuser, 2006.
3. J. B. Conway – Functions of one Variable, Narosa, New Delhi, 1996.
4. Lars V. Ahlfors – Complex Analysis, McGraw Hill, 3rd Edition, 1979.
5. 2. B. R. Ash – Complex Variables, Dover Publications, 2nd Edition, 2007.
6. R. V. Churchill, J. W. Brown and R. F. Verlag – Complex Variables and Applications, Mc Graw Hill, 8th Edition, 2009.

MTH 503	Measure and Integration	4 Credits (48 hours)

Course Objectives

1. **Understand Fundamental Concepts**: Gain a comprehensive understanding of the fundamental concepts in Lebesgue measure theory, including outer measures, σ-algebras, and measurable sets.

2. **Develop Analytical Skills**: Develop the ability to perform rigorous analysis using concepts such as countable additivity, continuity, and key theorems like the Borel-Cantelli lemma and Egoroff's theorem.
3. **Master Integration Techniques**: Achieve proficiency in the Riemann and Lebesgue integration techniques, understanding their applications and distinctions, and mastering the integration of various classes of functions.
4. **Explore Advanced Topics**: Explore advanced topics such as differentiation and integration of monotone functions, functions of bounded variation, and the properties and completeness of L^p spaces.

Course Outcomes

Upon successful completion of this course, students will gain the following :

1. Ability to define and work with Lebesgue measurable sets and functions, including performing operations such as sums, products, compositions, and approximations.
2. Proficiency in computing the Lebesgue integral for different types of functions, including bounded measurable functions and non-negative functions over sets of finite measure.
3. Understanding and application of advanced theorems related to integration, such as the Vitali convergence theorem, and the ability to distinguish between Riemann and Lebesgue integrability.
4. Ability to analyze the continuity and differentiability of monotone functions, work with functions of bounded variation, and understand the process of integrating derivatives and differentiating indefinite integrals.
5. Comprehension of the structure and properties of L^p spaces, including norms, inequalities, and the completeness of these spaces, and the ability to apply these concepts to solve relevant mathematical problems.

Unit I

Introduction to Lebesgue measure, Lebesgue outer measure, The σ-algebra of Lebesgue measurable sets, Outer and inner approximation of Lebesgue measurable sets, Countable additivity, continuity and the Borel-Cantelli lemma, (12 Hours)

Unit II

Non-measurable stes, Lebesgue measurable functions- Sums, products and compositions, Sequential point-wise limits, Simple approximation, Littilewood's three principles, Egoroff's theorem, Lusin's theorem, Riemann integral, Lebesgue integral of a bounded measurable function over a set of finite measure. (12 Hours)

Unit III

Lebesgue integral of a measurable non-negative function, General Lebesgue integral, Countable additivity and continuity of integration, Uniform integrability: The Vitali convergence theorem, Charaterizations of Riemann and Lebesgue integrability. (12 Hours)

Unit IV

Differentiation Integration: Continuity of monotone functions, Differentiability of monotone functions, Functions of bounded variation, Absolute continuity, Integrating derivatives: Differentiating indefinite integrals.

The L^p Spaces: Normed linear spaces, the inequalities of Young, Holder and Monkowski, Completeness of L^p Spaces. (12 Hours)

References

[1] H. L. Royden and P. M. Fitzpatrick, *Real Analysis*, 4th Ed., Pearson, 2015.

[2] H. L. Royden, *Real Analysis*, 3rd Ed., Prentice - Hall, 2003.

[3] G. D. Barra, *Introduction to Measure Theory*, Van Nostrand Reinhold Company Ltd., 1974.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., Tata McGraw Hill Publishing Company, 1987.

[5] P. R. Halmos, *Measure Theory*, Springer Verlag, 1974.

[6] F. Hewitt and K. Stromberg, *Real and Abstract Analysis*, Springer Verlag, 1965.

[7] Inder K. Rana, *An Introduction to Measure and Integration*, 2nd Ed., Narosa Publishing House, 1997.

MTH 504	Multivariate Calculus and Geometry	4 Credits (48 hours)

Course Objectives

1. **Understanding Differentiable Functions**: Develop a deep understanding of differentiable functions, including the concepts of level sets, tangent spaces, and optimization techniques like Lagrange multipliers.
2. **Exploring Curves and Their Geometry**: Gain insights into the geometry of curves in R^3, including the study of line integrals and the Frenet-Serret equations.
3. **Mastering Multiple Integrals**: Achieve proficiency in double and triple integrals, including their applications in Green's theorem, Stoke's theorem, and the divergence theorem.
4. **Studying Surfaces and Their Properties**: Understand the geometry of surfaces in R^3, including parametrized surfaces, surface area, surface integrals, Gaussian curvature, and geodesics.

Course Outcomes

Upon successful completion of this course, students will gain the following:

1. Ability to apply optimization techniques, including Lagrange multipliers, to find maxima and minima on open sets.
2. Proficiency in understanding and analyzing curves in R^3 using line integrals and the Frenet-Serret equations, and interpreting the geometric properties of these curves.
3. Ability to compute double and surface integrals and apply Green's theorem and Stoke's theorem to solve problems involving parametrized surfaces in R^3.
4. Competence in evaluating triple integrals and applying the divergence theorem, and understanding their geometric interpretations.
5. Understanding the geometry of surfaces in R^3, including concepts like Gaussian curvature and geodesics, and their applications in various mathematical and physical contexts.

Unit I

Introduction to differentiable functions-, Level sets and tangent spaces, Lagrange multipliers, Maxima and minima on open sets. (12 Hours)

Unit II

Curves in R^3, Line Integrals, The Frenet-Serret equations, Geometry of curves in R^3.
(12 Hours)

Unit III

Double integration - Green's theorem, Parametrised surfaces in R^3, Surface area, Surface integrals, Stoke's theorem. (12 Hours)

Unit IV

Triple integrals, The divergence theorem, The geometry of surfaces in R^3, Gaussian Curvature, Geodesic (8 Hours)

References

[1] Sean Dineen, *Multivariate Calculus and Geometry*, 3rd Ed., Springer Undergraduate Mathematics Series, 2014.

[2] Andrew Pressly, *Elementary Differential Geometry*, 2nd Ed., Springer Undergraduate Mathematics Series, 2010.

[3] Walter Rudin, *Principles of Mathematical Analysis*, 3rd Ed., McGraw Hill, New York, 1976.

[4] J. A. Thorpe, *Elementary Topics in Differential Geometry*, Undergraduate Texts in Mathematics, Springer Verlag, 1994.

[5] W. Klingenberg, *A course in Differential Geometry*, Springer Verlag, 1983.

MTS 505	Advanced Numerical Analysis	4 Credits (48 hours)

Course Objectives

1. **Master Numerical Integration**: Gain a comprehensive understanding of various numerical integration techniques, including those based on interpolation and undetermined coefficients, and specialized methods such as Romberg integration and Gauss quadrature methods.
2. **Solve Ordinary Differential Equations Numerically**: Develop proficiency in numerical methods for solving ordinary differential equations (ODEs), including Euler, Taylor series, Runge-Kutta, and multistep methods, as well as methods for boundary value problems.
3. **Analyze Systems of Linear Differential Equations**: Understand and apply numerical techniques for solving systems of linear differential equations, including finite difference methods and explicit schemes for different types of partial differential equations.
4. **Implement and Analyze Algorithms**: Enhance skills in implementing numerical algorithms and analyzing their accuracy, stability, and efficiency, particularly in the context of solving differential equations and integration problems.

Course Outcomes

1. Ability to implement and apply various numerical integration methods, including Romberg, Gauss-Legendre, Gauss-Chebyshev, Gauss-Laguerre, and Gauss-Hermite methods, as well as double integration using the trapezoidal and Simpson's rules.
2. Proficiency in solving ordinary differential equations using numerical methods such as Euler, Backward Euler, Mid-point, Taylor series, and Runge-Kutta methods, and addressing boundary value problems using finite difference and predictor-corrector methods.

3. Competence in solving systems of linear differential equations using difference methods for parabolic, hyperbolic, and elliptic equations, and understanding the application of schemes like Schmidt, Du Fort-Frankel, Crank-Nicolson, and Crandall.
4. Skill in implementing and analyzing numerical algorithms for differential equations and integration, with a focus on accuracy, stability, and computational efficiency.
5. Enhanced problem-solving abilities in applying numerical methods to practical and theoretical problems involving integration and differential equations, and the ability to critically evaluate and choose appropriate methods for specific problems.

Contents

Unit I - Numerical Integration:

Recapitulation of the methods based on interpolation, Methods based on undetermined coefficients. Romberg integration, Gauss-Legendre integration methods, Gauss-Chebyshev integration methods, Gauss-Laguerre integration methods, Gauss-Hermite integration methods. Double integration, Trapezoidal rule, Simpson's rule. (15 Hours)

Unit II - Ordinary Differential Equations:

Introduction, Numerical methods, Euler method, Backward Euler method, Mid-point method, Single step methods, Taylor series method, Runge-Kutta methods, Multistep methods, Determination of a_j and b_j, Predictor-corrector methods, Boundary value problems, Finite difference methods, Trapezoidal, Dahlquist and Numerov methods. (15 Hours)

Unit III - Systems of Linear Differential Equations:

Introduction, Difference methods, Parabolic equations in one space dimension, Schmidt formula, Du Fort-Frankel scheme, Crank-Nicolson and Crandall schemes, Solution of hyperbolic equation in one dimension by explicit schemes, The CFL condition, Elliptic equations, Dirichlet problem, Neumann problem, Mixed problem. (18 Hours)

References

[1] M. K. Jain, S. R. K. Iyengar, P. K. Jain, *Numerical Methods for Scientific and Engineering Computation*, 6th Ed., New Age International, 2012.

[2] C. F. Gerald and P. O. Wheatly, *Applied Numerical Analysis*, Pearson Education, Inc., 1999.

[3] M. K. Jain, *Numerical Solution of Differential Equations*, 2nd Ed., New Age International (P) Ltd., New Delhi, 1984.

[4] A. R. Mitchell, *Computational Methods in Partial Differential Equations*, John Wiley and Sons, Inc., 1969.

MTS 506	**Commutative Algebra**	**4 Credits (48 hours)**

Course Objectives

1. **Understand Rings and Ideals:** Develop a solid understanding of the fundamental concepts in ring theory, including zero divisors, nilpotent elements, units, and various types of ideals (prime, maximal, nilradical, Jacobson radical).

2. **Explore Modules:** Gain knowledge of module theory, including operations on submodules, isomorphism theorems, direct sums and products, finitely generated modules, and exact sequences.
3. **Analyze Modules of Fractions and Primary Decomposition:** Understand the local properties of modules, the concepts of extended and contracted ideals in rings of fractions, and the primary decomposition in module theory.
4. **Study Integral Dependence and Chain Conditions:** Learn about integral dependence, integrally closed integral domains, the going-up and going-down theorems, Noetherian rings and modules, and primary decomposition in Noetherian rings.

Course Outcomes

1. Ability to identify and work with zero divisors, nilpotent elements, units, prime ideals, and maximal ideals, and understand the concepts of the nilradical and Jacobson radical.
2. Proficiency in performing operations on ideals, including extensions and contractions, and applying these concepts to solve problems in ring theory.
3. Understanding and application of operations on submodules, isomorphism theorems, direct sums and products, and finitely generated modules, and the ability to utilize Nakayama's lemma and exact sequences in module theory.
4. Competence in analyzing local properties of modules, extended and contracted ideals in rings of fractions, and applying the first and second uniqueness theorems to practical problems.
5. Knowledge of integral dependence, integrally closed integral domains, and the going-up and going-down theorems, along with an understanding of Noetherian rings and modules and the ability to perform primary decomposition in Noetherian rings.

Contents

Unit I - Rings and Ideals:

Zero divisors, Nilpotent elements, Units, Prime ideals and maximal ideals, Nilradical and Jacobson radical, Operations on ideals, Extensions and contraction of ideals. (18 Hours)

Unit II - Modules:

Recapitulation of Operations on submodules, Isomorphism theorems. Direct sum and product, Finitely generated modules, Nakayama's lemma, Exact sequences (omit tensor products and related results). (12 Hours)

Unit III –Modules of Fractions and Primary Decomposition,:

Local properties, Extended and contracted ideals in rings of fractions, First and second uniqueness theorems. (12 Hours)

Unit IV - Integral Dependence and Chain Conditions:

Integral dependence, The going-up theorem, Integrally closed integral domains, The going-down theorem, Noetherian rings and modules, Primaryde composition in Noetherian rings.

(6 Hours)

References

[1] M. F. Atiyah and I. G. Macdonald, *Introduction to Commutative Algebra*, Indian Ed., Lavant Books, 2007.

[2] N. Bourbaki, *Commutative Algebra*, American Mathematical Society, 1972.

[3] N. S. Gopalkrishnan, *Commutative Algebra*, 2nd Ed., University Press, 2015.

[4] G. Northcott, *Lesson on Rings, Modules and Multiplicities*, Cambridge University Press, 2008.

[5] O. Zariski and P. Samuel, *Commutative Algebra* Vol I, II, Graduate Texts in Mathematics, Springer Verlag, 1976.

MTS 508	Lattice Theory	4 Credits (48 hours)

Course Objectives

1. **Understand Posets**: Grasp the fundamentals of partially ordered sets, diagrams, bounds, homomorphisms, and isomorphisms. Learn Zorn's lemma and the Hausdorff maximal chain principle.
2. **Explore Lattice Theory**: Study lattices as posets and algebraic structures, including duality, semilattices, sublattices, and various complemented lattices.
3. **Analyze Complete Lattices**: Understand complete lattices, fixed point property, compact elements, distributive and modular lattices, and key theorems.
4. **Learn Boolean Algebras**: Study complemented modular lattices, uniquely complemented lattices, Boolean algebras, De Morgan formulae, and their relationship to Boolean rings and fields of sets.

Course Outcomes:

At the end of the course students will be able to

1. Define and work with posets, diagrams, bounds, homomorphisms, and understand Zorn's lemma and Hausdorff principle.
2. Identify and work with semilattices, sublattices, atomic lattices, ideals, prime ideals, and apply the homomorphism theorem.
3. Analyze complete lattices, understand fixed point property, compact elements, and characterize distributive and modular lattices.
4. Work with complemented lattices, bounded lattices, Boolean algebras, apply De Morgan formulae, and understand Boolean rings and fields of sets.
5. Apply theoretical concepts to practical problems in posets, lattices, and Boolean algebras.

Contents

Unit I - Partially Ordered Sets:

Partially ordered sets (or Posets), Diagrams, Lower and upper bounds, Order homomorphism and order isomorphism, Special subsets of a poset. Axiom of choice (Statement only). Zorn's lemma and Hausdorff's maximal chain principle, and proof of the equivalence of these two statements. Length of a poset, The minimum and maximum conditions, Duality principle for posets.

(12 Hours)

Unit II - Lattices in General:

A lattice as a poset and as an algebra, Diagrams of lattices, Duality principle for lattices, Semilattices, Sublattices, Ideals and prime ideals of lattices, Ideal generated by a nonempty subset of a lattice and its description, The ideal lattice and the augmented ideal lattice of a lattice, Bound elements, atoms and dual atoms in a lattice, Atomic lattices, complemented, relatively complemented and sectionally complemented lattices, Homomorphisms, congruence relations and quotient lattices of lattices, The homomorphism theorem. (12 Hours)

Unit III - Complete Lattices, Distributive and Modular Lattices:

Complete lattices, fixed point property. Compact elements and compactly generated lattices. Distributive, Modular lattices, Characterizations of modular and distributive lattices in terms of sublattices, The isomorphism theorem of modular lattices, The prime ideal theorem for distributive lattices. (18 Hours)

Unit V - Complemented Modular Lattices and Boolean Algebras:

Complemented modular lattices and bounded relatively complemented lattices. Distributivity of a uniquely complemented relatively complemented lattice, Boolean algebras, De Morgan formulae, Boolean algebras and Boolean rings, Distributive lattices and rings of sets, Boolean algebras and fields of sets. (6 Hours)

References

[1] G. Szasz, *Introduction to Lattice Theory*, Academic Press, N.Y., 1963.

[2] G. Gratzer, *General Lattice Theory*, Birkhauser Verlag, Basel, 1978.

[3] P. Crawley and R.P, Dilworth, *Algebraic Theory of Lattices*, Prentice - Hall Inc., N. J., 1973.

[4] G. Birkho , *Lattice Theory*, American Mathematical Society Colloquium Publications, Volume 25, 1995.

[5] L. A. Skornjakov, *Elements of Lattice Theory*, Hindustan Publishing Corporation, 1977.

MTS 513	Applied Algebraic Coding theory	4 Credits (48 hours)

Course Objectives

1. **Understand Fundamentals**: Introduce the fundamental concepts and importance of coding theory in digital communication and storage.
2. **Explore Linear and Cyclic Codes**: Study the structure, properties, and decoding techniques of linear and cyclic codes.
3. **Delve into Advanced Codes**: Learn about BCH, Reed-Solomon, and convolutional codes, their construction, and decoding algorithms.
4. **Apply Theoretical Knowledge**: Explore applications of coding theory in real-world scenarios and understand recent advancements in the field.

Course Outcomes

1. Understand and explain the basic concepts of coding theory and its importance in digital communication.

2. Utilize the properties of linear and cyclic codes in error detection and correction.
3. Construct and decode BCH, Reed-Solomon, and convolutional codes for various applications.
4. Understand and apply LDPC and turbo codes in advanced communication systems.
5. Keep abreast of recent developments in coding theory and apply theoretical knowledge to practical problems.

Contents

Unit I: Introduction to Coding Theory and Linear Codes (12 Hours)

Introduction to Coding Theory: Basic concepts of coding theory, Importance and applications of coding theory in digital communication and storage. Linear Codes: Definitions and properties of linear codes, Generator and parity-check matrices, Encoding and decoding of linear codes, Hamming distance and error-detecting/correcting capability. Standard Array and Syndrome Decoding: Standard array for a linear code, Syndrome decoding techniques, Error detection and correction using syndromes.

Unit II: Cyclic Codes and BCH Codes (12 Hours)

Cyclic Codes: Definition and properties of cyclic codes, Polynomial representation of cyclic codes, Generator and parity-check polynomials, Encoding and decoding of cyclic codes. BCH Codes: Definition and construction of BCH codes, Minimum distance and error-correcting capability of BCH codes, Decoding BCH codes using the Berlekamp-Massey algorithm.

Unit III: Finite Fields and Reed-Solomon Codes (12 Hours)

Finite Fields: Introduction to finite fields and their properties, Construction and arithmetic of finite fields, Applications of finite fields in coding theory. Reed-Solomon Codes: Definition and construction of Reed-Solomon codes, Properties and applications of Reed-Solomon codes, Encoding and decoding algorithms for Reed-Solomon codes.

Unit IV: Advanced Topics in Coding Theory (12 Hours)

Convolutional Codes: Definition and properties of convolutional codes, State diagrams and trellis representations, Viterbi algorithm for decoding convolutional codes. LDPC and Turbo Codes: Introduction to LDPC (Low-Density Parity-Check) codes, Encoding and decoding of LDPC codes, Introduction to turbo codes and their decoding algorithms. Applications and Recent Developments: Applications of coding theory in modern communication systems, Recent developments and advanced topics in coding theory.

References

[1] F. J. MacWilliams and N. J. A. Sloane, *The Theory of Error-Correcting Codes*, North-Holland, 1977.

[2] S. Lin and D. J. Costello, *Error Control Coding: Fundamentals and Applications*, Pearson, 2nd Edition, 2004.

[3] W. Cary Huffman and Vera Pless, *Fundamentals of Error-Correcting Codes*, Cambridge University Press, 2003.

[4] R. E. Blahut, *Algebraic Codes for Data Transmission*, Cambridge University Press, 2002.

[5] Tom Richardson and Ruediger Urbanke, *Modern Coding Theory*, Cambridge University Press, 2008.

MTS 514	Operations Research	4 Credits (48 hours)

Course Objectives

1. **Understand Operations Research**: Introduce the scope, phases, and models of Operations Research, and their applications in decision-making.
2. **Master Linear Programming**: Develop skills in formulating and solving Linear Programming Problems (LPP) using graphical and simplex methods.
3. **Explore Advanced Techniques**: Learn advanced methods like the Simplex Method, Dual Simplex Method, and Sensitivity Analysis.
4. **Apply Optimization Methods**: Solve transportation and assignment problems using optimization techniques, and understand Integer Programming Problems.

Course Outcomes

1. Explain the fundamentals and importance of Operations Research in decision-making processes.
2. Formulate and solve linear programming problems using various methods, including graphical, simplex, and dual simplex.
3. Analyze the sensitivity of solutions to changes in parameters and resolve issues like degeneracy and unbounded solutions.
4. Optimize transportation and assignment problems using techniques like the North-West Corner Rule, VAM, MODI Method, and Hungarian Method.
5. Apply integer programming methods and solve problems using the Cutting-Plane Method and Gomory's Fractional Cut Algorithm.

Contents

Unit I - Operations Research and Modeling with Linear Programming:
Basics of Operations Research: Introduction, Scope of Operations Research, Phases of Operations Research. Models in Operations Research, Classification of Models, Uses and Limitations of Operations Research. Operations Research and Decision-making.
Linear Programming: Formulation of LP problems, Sensitivity Analysis, Procedure for Solving LPP by Graphical Method, General Formulation of LPP, Matrix Form of LPP, Canonical or Standard Forms of LPP. Simplex Method-Simplex Algorithm. (12 Hours)

Unit II - The Simplex Method and Sensitivity Analysis:

Artificial Variables Technique: The Charne's Big-*M* Method, The Two-phase Simplex Method, Degeneracy, Methods to Resolve Degeneracy, Unbounded Solution.
Duality in Linear Programming: Formation of Dual Problems, Definition of the Dual Problem, Important Results in Duality, Dual Simplex Method, Dual Simplex Algorithm.
Revised Simplex Method: Computational procedure. (12 Hours)

Unit III - Duality and Post-Optimal Analysis:

Transportation Problem: Mathematical Formulation, Definitions, Optimal Solution, North-West Corner Rule, Least Cost or Matrix Minima Method, Vogel's Approximation Method (VAM), Optimality Test, MODI Method, The Stepping-Stone Method.
Transshipment : Transhipment Problem, Definitions, Transhipment Problem-to-Transportation Problem, Transhipment Model. (12 Hours)

Unit IV - Transportation Model and Its Variants:

Assignment Problem: Definition, Mathematical Formulation of an Assignment Problem, Difference between Transportation and Assignment Problems , Hungarian Method Procedure, Unbalanced Assignment Problem, Maximization in Assignment Problem.
Integer Programming Problems: Importance of Integer Programming Problems, Applications of Integer Programming, Methods of Integer Programming Problem, Cutting-Plane Method, Search Method, Gomory's Fractional Cut Algorithm. (12 Hours)

References

[1] Hamdy A Taha, *Introduction to Operation Research*, Tenth Ed., Pearson Education Limited, 2017.

[2] F. S. Hillier, G.J. Lieberman, *Introduction to Operations Research*, Concepts and Cases, 8th Edition, 2010, TMH

[3] P. Ramamurthy, *Operations Research*, New Age International, 2007.

[4] J. K. Sharma, *Operations Research*- Theory and Applications, Macmillan Publishers, fourth edition 2009.

MTS 516	Design and Analysis of Algorithms	4 Credits (48 hours)

Course Objectives

1. To introduce the fundamental concepts of algorithm design and analysis.
2. To study various algorithmic techniques for problem-solving.
3. To develop the ability to analyze the time and space complexity of algorithms.

4. To explore the application of algorithms in various domains of mathematics and computer science.

Course Outcomes:

1. Understand and apply various algorithm design techniques to solve problems.
2. Analyze the efficiency of different algorithms.
3. Implement algorithms and validate their correctness and performance.
4. Apply algorithms in real-world scenarios and mathematical problem-solving.
5. Develop an understanding of advanced topics in algorithm design and analysis.

Contents

Unit I: Introduction to Algorithms and Complexity Analysis (12 Hours)
Introduction to Algorithms: Definition, importance, and applications of algorithms in computer science and mathematics. Complexity Analysis: Time and space complexity, Big O, Omega, and Theta notations, asymptotic analysis. Recurrence Relations: Methods to solve recurrences, Master Theorem, and its applications. Divide and Conquer: Basic principles, examples such as Merge Sort, Quick Sort, and Binary Search.

Unit II: Algorithm Design Techniques (12 Hours)
Greedy Algorithms: Principles, examples such as Huffman coding, Prim's algorithm, and Kruskal's algorithm. Dynamic Programming: Principles, examples such as the Fibonacci sequence, Matrix Chain Multiplication, and the Knapsack problem. Backtracking and Branch-and-Bound: Principles, examples such as N-Queens problem, and solving combinatorial optimization problems.

Unit III: Graph Algorithms (12 Hours)
Graph Representations: Adjacency matrix, adjacency list. Graph Traversal: Depth-first search (DFS), Breadth-first search (BFS). Shortest Path Algorithms: Dijkstra's algorithm, Bellman-Ford algorithm, Floyd-Warshall algorithm. Minimum Spanning Tree Algorithms: Prim's algorithm, Kruskal's algorithm.

Unit IV: Advanced Topics in Algorithms (12 Hours)
NP-Completeness: P, NP, NP-hard, NP-complete classes, Cook-Levin theorem, and examples of NP-complete problems. Approximation Algorithms: Introduction, approximation ratio, Examples such as Vertex Cover and Traveling Salesman Problem. Randomized Algorithms: Principles, Examples such as Randomized Quick Sort and Monte Carlo algorithms. Parallel Algorithms: Introduction to parallel computing, Examples and parallel matrix multiplication.

References:

1. Cormen, T.H., Leiserson, C.E., Rivest, R.L., and Stein, C., "*Introduction to Algorithms*", MIT Press, 3rd Edition, 2009.
2. Kleinberg, J., and Tardos, E., "*Algorithm Design*", Pearson, 1st Edition, 2005.
3. Dasgupta, S., Papadimitriou, C.H., and Vazirani, U., "*Algorithms*", McGraw-Hill Education, 1st Edition, 2008.
4. Aho, A.V., Hopcroft, J.E., and Ullman, J.D., "*The Design and Analysis of Computer Algorithms*", Addison-Wesley, 1st Edition, 1974.

5. Sedgewick, R., and Wayne, K., "*Algorithms*", Addison-Wesley, 4th Edition, 2011.
6. Manber, U., "*Introduction to Algorithms: A Creative Approach*", Addison-Wesley, 1st Edition, 1989.
7. Garey, M.R., and Johnson, D.S., "*Computers and Intractability: A Guide to the Theory of NP-Completeness*", W.H. Freeman, 1st Edition, 1979.

MTS 516	Advanced Number Theory	4 Credits (48 hours)

Course Objectives

1. **Introduce Fundamental Concepts**: Provide a comprehensive understanding of arithmetical functions, congruences, and their fundamental properties.
2. **Explore Advanced Number Theory**: Examine advanced topics such as quadratic residues, primitive roots, and their applications in solving complex number theory problems.
3. **Understand Partition Theory**: Delve into the elementary theory of partitions, generating functions, and their combinatorial proofs, enhancing problem-solving skills.
4. **Study q-series and Theta Functions**: Introduce q-series and theta functions, exploring their fundamental theorems and applications, including Ramanujan's congruences.

Course Outcomes:

At the end of the course students will be able to

1. Gain a thorough understanding of arithmetical functions, including the Mobius and Euler functions, and their applications in mathematical problems.
2. Apply the basic properties of congruences, including linear congruences and the Euler-Fermat theorem, to solve number theory problems.
3. Understand and apply the quadratic reciprocity law, Legendre's symbol, and Euler's criterion to solve Diophantine equations.
4. Utilize the elementary theory of partitions, generating functions, and combinatorial proofs to solve advanced mathematical problems.
5. Comprehend the basics of q-series and theta functions, including fundamental theorems and elementary congruences, and apply these concepts to advanced mathematical contexts.

Contents

Unit I

Arithmetical Functions: The Mobius function and its properties, Euler function, examples and properties, The Dirichlet product of arithmetical functions, Dirichlet inverses and the Mobius inversion formula.
Congruences: Recapitulation of basic properties of congruences, Residue classes and complete residue systems, Linear congruences. Reduced residue systems and the Euler-Fermat theorem.

(12 Hours)

Unit II

Quadratic Residues, Quadratic Reciprocity Law: Quadratic residues, Legendre's symbol and its properties, Euler's criterion, Gauss lemma, The quadratic reciprocity law and its applications, The Jacobi symbol, Applications to Diophantine equations.

Primitive Roots: The exponent of a Number mod m, Primitive roots and reduced residue systems. The non-existence of primitive roots mod 2^{α} for $\alpha \geq 3$, The primitive roots mod p for odd primes p, Primitive roots and quadratic Residues. (12 Hours)

Unit III

Partitions - Elementary theory of Partitions, Graphical representation of partitions, The generating function of $p(n)$, other generating functions, Two theorems of Euler, Jacobi's Triple product identity and its applications, combinatorial proofs of Euler's identity, Euler's pentagonal number theorem, Franklin's combinatorial proof. (12 Hours)

Unit IV

Notations and Arithmetical functions, Introduction to q-series and Theta function, Fundamental theorems about q-series and Theta function, Elementary congruences for $p(n)$ and $\tau(n)$, Ramanujan's congruence $p(5n+4) \equiv 0(mod\ 5)$ and $p(7n+5) \equiv 0(mod\ 7)$, The parity of $p\ (n)$. (12 Hours)

References

[1] Tom M. Apostol, *Introduction to Analytic Number Theory*, Springer, 1989.

[2] G. E. Andrews, *The Theory of Partitions*, Addison Wesley, 1976.

[3] Bruce C. Berndt, *Number theory in the spirit of Ramanujan*, American Mathematical Society, 2006.

[4] G. H. Hardy and E. M. Wright, *An Introduction to Theory of Numbers*, 5th Ed., Oxford University Press, 1979.

[5] David M. Burton, *Elementary Number Theory*, 7th Ed., McGraw-Hill, 2010.

[6] I. Niven, H. S. Zuckerman and H. L. Montgomery, *An Introduction to the Theory of Numbers*, 5th Ed., New York, John Wiley and Sons, Inc., 2004.

[7] Bruce C. Berndt, Ramanujan's Note Books Volumes-1 to 5.

[8] A. K. Agarwal, Padmavathamma, M. V. Subbarao, *Partition Theory*, Atma Ram & Sons, Chandigarh, 2005.

MTS 517	**Classical Mechanics**	**4 Credits (48 hours)**

Course *Objectives*

1. To develop a thorough understanding of the principles of classical mechanics and their mathematical formulations.
2. To equip students with the skills to solve complex mechanical problems using Newtonian, Lagrangian, and Hamiltonian mechanics.
3. To prepare students for advanced topics in theoretical physics and applied mathematics, particularly for competitive examinations.

4. To enhance analytical and problem-solving abilities relevant to research and academia in physics and mathematics.

Course Outcomes:

1. Students will gain a deep understanding of Newtonian mechanics, including the laws of motion, conservation principles, and central force problems.
2. Students will be able to apply variational principles and Lagrangian mechanics to solve complex mechanical systems, understanding constraints and generalized forces.
3. Students will master Hamiltonian mechanics, including the formulation of Hamilton's equations, canonical transformations, and applications to physical systems.
4. Students will develop an understanding of advanced topics such as small oscillations, rigid body dynamics, special relativity, and chaos theory, and will be able to apply these concepts in various contexts.
5. Students will be well-prepared for further studies and research in theoretical physics and applied mathematics, as well as for competitive examinations.

Contents

Unit I: Newtonian Mechanics and Conservation Laws

Newton's Laws of Motion: Review of Newton's laws, inertial frames, equations of motion. Conservation Laws: Linear momentum, angular momentum, energy conservation; applications to collisions, rocket motion. Central Force Problems: Motion under central forces, orbits, Kepler's laws. Non-Inertial Frames: Pseudo forces, rotating frames of reference, Coriolis and centrifugal forces. (12 Hours)

Unit II: Lagrangian Mechanics

Variational Principles: Action, principle of least action, Hamilton's principle. Lagrangian Formulation: Generalized coordinates, Lagrange's equations of the first and second kinds. Applications: Simple pendulum, double pendulum, central force problems. Constraints and Generalized Forces: Types of constraints, D'Alembert's principle, virtual work. (12 Hours)

Unit III: Hamiltonian Mechanics (12 Hours)

Hamiltonian Formulation**:** Hamilton's equations of motion, phase space, physical significance. Canonical Transformations: Generating functions, Poisson brackets, properties and applications. Applications: Harmonic oscillator, central force problems, planetary motion. Liouville's Theorem: Conservation of phase space volume, applications in statistical mechanics. (12 Hours)

Unit IV: Advanced Topics in Classical Mechanics (12 Hours)

Theory of Small Oscillations: Normal modes, stability analysis, coupled oscillators. Rigid Body Dynamics: Euler's equations, rotating rigid bodies, gyroscopic motion, moments of inertia. Special Relativity: Lorentz transformations, relativistic mechanics of particles, energy and momentum. Nonlinear Dynamics and Chaos: Introduction to chaos theory, examples of chaotic systems, Lyapunov exponents. (12 Hours)

References:

[1] Goldstein, Herbert, Poole, Charles P., and Safko, John L., Classical Mechanics, Pearson, 3rd Edition, 2001.
[2] Marion, Jerry B., and Thornton, Stephen T., Classical Dynamics of Particles and Systems, Cengage Learning, 5th Edition, 2003.
[3] Landau, L.D., and Lifshitz, E.M., Mechanics: Volume 1, Pergamon Press, 1st Edition, 1976.
[4] Morin, David, Introduction to Classical Mechanics, Cambridge University Press, 1st Edition, 2008.
[5] Taylor, John R., Classical Mechanics, University Science Books, 1st Edition, 2005.
[6] Marsden, Jerrold E., and Ratiu, Tudor S., Introduction to Mechanics and Symmetry, Springer, 2nd Edition, 1999.
[7] Susskind, Leonard, and Hrabovsky, George, Classical Mechanics: The Theoretical Minimum, Basic Books, 1st Edition, 2013.
[8] Strogatz, Steven H., Nonlinear Dynamics and Chaos, Westview Press, 1st Edition, 1994.

MTL 517	Practical - III	2 Credits (2 hours lab /week)

Mathematics Practicals using Python Programming Language.

Course Objectives

1. **Develop Practical Programming Skills:** Equip students with the ability to implement mathematical algorithms in Python, emphasizing numerical differentiation, integration, and solving differential equations.
2. **Understand Numerical Methods:** Provide a hands-on understanding of various numerical methods, such as Trapezoidal and Simpson's rules, Euler's methods, and the Runge-Kutta method.
3. **Apply Mathematical Concepts:** Enable students to apply mathematical concepts such as differentiation, integration, and Jacobians in practical programming scenarios.
4. **Visualization and Animation:** Teach students how to visualize and animate mathematical curves and solutions to enhance understanding and interpretation of results.
5. **Analytical and Computational Skills:** Foster analytical and computational skills necessary for solving complex mathematical problems and interpreting the results through programming.

Course Outcomes

Upon completing this lab course, students will be able to:

1. Create animations of plotted mathematical curves, enhancing the visual understanding of mathematical functions and their behaviors.
2. Develop Python programs to perform numerical differentiation, understanding the importance and application of numerical methods.
3. Write Python programs to evaluate integrals using various numerical methods such as the Trapezoidal rule and Simpson's rules.
4. Apply Picard's method, Euler's methods, and the Runge-Kutta method to solve differential equations and initial value problems.

5. Develop programs to find extreme values of functions of several variables, using concepts of calculus.
6. Implement algorithms to find the length of given space curves, applying knowledge of parametric equations and calculus.

List of Programs

1) Program to animate the plotted curves.
2) Programmes on Numerical Differentiation.
3) Program to evaluate the given integral using Trapezoidal rule/ Simpson's 1/3 rule/Simpson's 3/8 rule.
4) Program to find the approximate solution of a differential equation with initial condition by Picard's method of successive approximation
5) Program to solve an initial value problem using Euler's Method/ Euler's Modified Method.
6) Program to solve an initial value problem using Fourth Order Ruge-Kutta Method.
7) Program to find extreme values of functions of a several variables.
8) Program to find the length of a given space curve.
9) To find the Derivative, Partial Derivative and Jacobian of functions of a several variables.
10) To evaluate given multiple integrals.
11) Program to find Area and Volume of surfaces/regions/solids using Integration.
12) Solving Differential equations and plotting the solutions (Analytical Methods)

Note: The above list may be changed annually with the approval of the PG BOS in Mathematics.

IV Semester

MTH 552	Complex Analysis -II	4 Credits (48 hours)

Course Objectives

1. To provide an in-depth understanding of advanced topics in complex analysis, focusing on conformal mapping, Mobius maps, and their applications.
2. To explore and apply the fundamental theorems and principles such as the maximum modulus principle, Schwarz' lemma, and Liouville's theorem.
3. To study the properties and classifications of singularities and meromorphic functions, including the essential singularities and Picard's theorem.
4. To delve into advanced topics such as the residue theorem, analytic continuation, infinite products, and special functions like the Gamma and Zeta functions.

Course Outcomes

1. Understand and apply the principle of conformal mapping and the properties of Mobius maps in complex analysis.

2. Utilize the maximum modulus principle, Schwarz' lemma, and Liouville's theorem to solve complex analysis problems.
3. Analyze and classify singularities and meromorphic functions, and apply the residue theorem and Rouche's theorem in complex function theory.
4. Apply advanced concepts such as analytic continuation, infinite products, and special functions in solving complex analysis problems, including understanding the order and genus of entire functions.
5. Competence in applying advanced theorems and functions in complex analysis, including canonical products and gamma functions.

Contents

Unit I

Principle of conformal mapping, Basic properties of Mobius maps, Fixed points and Mobius maps, The cross-ratio and its invariance property, Maximum modulus principle, Hadamard's three circles/lines theorem. (12 Hours)

Unit II

Schwarz' lemma and its consequences, Liouville's theorem, Doubly periodic entire functions, Isolated and non-isolated singularities, Removable singularities, Poles, Illustration for poles through Laurent's series, Isolated singularities at infinity. (12 Hours)

Unit III

Meromorphic functions, Essential singularities and Picard's theorem, Residue at a finite point, Residue at the point at infinity, Residue theorem, Number of zeros and poles, Rouche's theorem.

(12 Hours)

Unit IV

Direct analytic continuation, Poisson integral formula, Infinite sums and meromorphic functions, Infinite product of complex numbers, Infinite product of analytic functions, Factorization of entire functions, The Gamma function, The Zeta function, Jensen's formula, The order and Genus of entire functions. (12 Hours)

References:

1. J. B. Conway – Functions of one Variable, Narosa, New Delhi, 1996.
2. Lars V. Ahlfors – Complex Analysis, McGraw Hill, 3rd Edition, 1979.
3. 2. B. R. Ash – Complex Variables, Dover Publications, 2nd Edition, 2007.
4. R. V. Churchill, J. W. Brown and R. F. Verlag – Complex Variables and Applications, Mc Graw Hill, 8th Edition, 2009.
5. S. Ponnuswamy and H. Silverman – Complex Variables with Applications, Birkhäuser, 2006.

References

[1] S. Ponnusamy - Foundations of Complex Analysis, Narosa, New Delhi, 2nd edition, 2005.

[2] Lars V. Ahlfors, *Complex Analysis*, 3rd Ed., McGraw Hill, 1979.

[3] S. Ponnuswamy and H. Silverman, *Complex Variables with Applications*, Birkauser, 2006.

[4] J. B. Conway, *Functions of one Variable*, Narosa, New Delhi, 1996.

[5] B. R. Ash, *Complex Variables*, 2nd Ed., Dover Publications, 2007.

[6] R. V. Churchill, J. W. Brown and R. F. Verlag, *Complex Variables and Applications*, 8th Ed., McGraw Hill, 2009.

MTH 553	Functional Analysis	4 Credits (48 hours)

Course Objectives:

1. **Understand Fundamental Concepts in Metric and Normed Spaces:** Explain the concepts of Baire's theorem, Normed linear spaces and provide examples. Identify and discuss properties of Banach spaces.
2. **Master Theorems and Fundamental Theorems in Functional Analysis:** Understand and utilize the Hahn-Banach theorem, the open mapping theorem, and the closed graph theorem in various contexts.
3. **Explore Advanced Topics in Functional Analysis:** Investigate the properties and applications of Hilbert spaces, including the parallelogram law and orthogonal complements.
4. **Develop Proficiency in Advanced Concepts of Hilbert Spaces and Operators:** Explore the properties of adjoint operators, normal and unitary operators, and projections. Apply the finite-dimensional spectral theorem and spectral resolution techniques.

Course Outcomes:

By the end of the course, students will be able to

1. Demonstrate a solid understanding of Baire's theorem, identifying and classifying normed linear spaces, distinguishing Banach spaces and their essential properties.
2. Apply foundational theorems such as the Heine-Borel theorem adapted to finite-dimensional normed spaces, utilize the Hahn-Banach theorem, open mapping theorem, and closed graph theorem to solve problems in functional analysis.
3. Demonstrate competence in applying the concepts of projections in Banach spaces and analyzing Hilbert spaces, understanding the parallelogram law, and effectively utilizing concepts of orthogonal complements.
4. Students will be able to explain the significance of orthonormal sets and computing adjoint operators, self-adjoint operators, normal and unitary operators, and applying spectral resolution techniques and the finite-dimensional spectral theorem.
5. Finally, students will develop enhanced problem-solving skills in advanced functional analysis, particularly in complex applications involving Hilbert spaces, operators, and spectral theory.

Contents

Unit I

Review of metric spaces: Convergence, Completeness and Baire's theorem.

Normed linear spaces: Definition and some examples, Banach spaces: Definition and some examples. Some basic properties, Quotient normed spaces. (12 Hours)

Unit II

Continuous linear transformations, Examples, Properties, Extension of Heine-Borel theorem to finite-dimensional normed spaces. The natural embedding of N in N^{**} , The Hahn Banach theorem, The open mapping theorem, Closed graph theorem. (12 Hours)

Unit III

Projections on Banach spaces, Uniform boundedness principle, Conjugate of an operator on a normed linear space.

Hilbert spaces:Definition and examples, Some basic properties, Parallelogram law, Orthogonal complements. (12 Hours)

Unit IV

Orthonormal sets, Complete Orthonormal sets. The conjugate of a Hilbert space, The Adjoint Operator, Self-adjoint operators, Normal and unitary operators, Projections, Finite dimensional spectral theorem, Spectral resolution. (12 Hours)

References

[1] G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw Hill, 2004.

[2] A. E. Taylor, David Lay, *Introduction to Functional Analysis*, John Wiley and Sons, 1980.

[3] Ward Cheney, *Analysis for Applied Mathematics*, Graduate Texts in Mathematics, Springer, 2001.

[4] Walter Rudin, *Real and Complex Analysis*, 3rd Ed., McGraw Hill, 1986.

[5] M. Thamban Nair, *Functional Analysis* - A First Course, Prentice-Hall, 2002.

MTS 554	Partial Differential Equations	4 Credits (48 hours)

Course Objectives

1. **Explore ODEs in Multiple Variables:** Recap methods for solving differential equations in more than two variables, including Pfaffian differential forms, and study orthogonal trajectories on surfaces.
2. **Understand First Order PDEs:** Learn the origins of first order partial differential equations, methods like Cauchy's characteristics and Charpit's method, and analyze integral surfaces and orthogonal surfaces.
3. **Study Higher Order PDEs:** Analyze linear PDEs with constant coefficients, classify second-order PDEs, and solve elliptic problems using methods for Laplace's equation in various coordinates.
4. **Examine Parabolic and Hyperbolic PDEs:** Investigate diffusion equations and solutions using separation of variables, Dirac Delta function, and study hyperbolic PDEs with methods like D'Alembert's solution and boundary value problems.

Course Outcomes

1. Ability to solve ordinary differential equations in multiple variables using Pfaffian forms and analyze orthogonal trajectories.
2. Proficiency in solving first order PDEs using Cauchy's method, Charpit's method, and understanding integral surfaces and orthogonality.
3. Skill in solving linear PDEs with constant coefficients, classifying second-order PDEs, and applying methods to elliptic differential equations.
4. Competence in solving diffusion equations, using Dirac Delta function, and applying separation of variables in cylindrical and spherical coordinates.
5. Ability to solve hyperbolic PDEs, including one-dimensional wave equations, using canonical reduction, D'Alembert's solution, and boundary value methods.

Unit I

Ordinary differential equations in more than two variables: Recapitulation of Methods of solution of $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$, Pfaffian differential forms and Pfaffian differential equations and solutions. Orthogonal trajectories of a system of curves on a surface. (12 Hours)

Unit II

First order partial differential equations: Origin of first order partial differential equations, The Cauchy problem for first order equations, Linear equations of first order, Integral surfaces passing through a given curve, Surfaces orthogonal to a given system of surfaces, Nonlinear equations of first order, Cauchy's method of characteristics, Charpit's method, Special types of first order equations. (12 Hours)

Unit III

Higher Order Partial Differential Equations: Linear partial differential equations with constant coefficients, Classification of second order PDE, Canonical forms, Adjoint operators, Riemann's method.

Elliptic Differential Equations: Dirichlet problem for a rectangle, Neumann problem for a rectangle, interior and exterior Dirichlet problem for a circle, Interior Neumann problem for a circle. Solution of Laplace equation in Cylindrical and Spherical coordinates. (12 Hours)

Unit IV

Parabolic Differential Equations: Occurrence of the diffusion equation, Elementary solutions of the diffusion equation, Dirac Delta function, Separation of variables, Solution of diffusion equation in Cylindrical and spherical coordinates.

Hyperbolic Differential Equations: Solution of one dimensional equation by canonical reduction, Initial value problem - D'Alembert's solution, Vibrating string - variable separation method, Forced vibrations, Boundary and initial value problems for two dimensional wave equation, Uniqueness of the solution for the wave equation, Duhamel's principle. (12 Hours)

References

[1] Ian Sneddon, *Elements of Partial Differential Equations*, International student Ed., Mc-Graw Hill, 1957.

[2] K. Sankara Rao, *Introduction to Partial Differential Equations*, Prentice-Hall of India, 1995.

[3] F. John, *Partial Differential Equations*, Springer Verlag, New York, 1971.

[4] P. Garabedian, *Partial Differential Equations*, Wiley, New York, 1964.

MTS 555	Advanced Topology	4 Credits (48 hours)

Course Objectives

1. **Understand Order Relations:** Study order relations, well-ordering theorem, minimal uncountable well-ordered sets, and topological properties of ordered spaces including box and product topologies.
2. **Explore Countability and Separation Axioms:** Analyze countability axioms, separation properties of spaces, and study the Urysohn Lemma, Imbedding theorem, and Urysohn Metrization theorem.
3. **Learn Metrization and Paracompactness:** Investigate the Nagata-Smirnov Metrization Theorem, explore local finiteness, and understand the concept of paracompactness in topological spaces.
4. **Study Fundamental Group and Covering Spaces:** Examine homotopy of paths, fundamental groups, and covering spaces, including the fundamental group of the circle.

Course Outcomes

1. Proficiency in understanding order relations, well-ordering theorem, and topological properties like least upper bounds and compactness in ordered sets.
2. Ability to apply countability and separation axioms, and utilize the Urysohn Lemma and Metrization theorems in various spaces.
3. Competence in applying the Nagata-Smirnov Metrization Theorem, understanding local finiteness, and utilizing concepts of paracompactness.
4. Skill in analyzing homotopy, fundamental groups, and covering spaces, with specific focus on the fundamental group of the circle.
5. Ability to apply advanced topological concepts such as compactness, separation properties, and covering spaces in problem-solving and theoretical analysis.

Unit I - Preliminaries (12 Hours)

Order relations and dictionary order relations, Well ordering theorem, The minimal uncountable well ordered set S_Ω and its basic properties. The order topology and the ordered square I_o^2, the least upper bound property of I_o^2. Box and product topologies on arbitrary products of spaces and continuity of a function from a space into these products. Compact sets in ordered sets having the least upper bound property, Equivalence of compactness, limit point compactness and sequential compactness in metrizable spaces.

Unit II - Countability and separation axioms (12 Hours)

The Countability axioms and their properties, study of Countability properties of spaces R_l, R_l^2, I_o^2, S_Ω and $S_\Omega \times \overline{S_\Omega}$. The separation axioms and their properties, separation properties of spaces R_K, $S_\Omega$and $S_\Omega \times \overline{S_\Omega}$. Urysohn lemma(Statement only), Imbedding theorem and Urysohn Metrization theorem, Partitions of unity (finite case), Imbeddings of manifolds.

Unit III - Metrization theorems and para-compactness (12 Hours)
Local finiteness. The Nagata-Smirnov Metrization Theorem. Paracompactness.

Unit IV - The fundamental group and covering spaces (12 Hours)
Homotopy of paths, The fundamental group, Covering spaces, The fundamental group of the circle.

References

[1] J. R. Munkres*, Topology*, 2nd Ed., Pearson Education, Inc, 2000.

[2] G. F. Simmons, *Introduction to Topology and Modern Analysis*, Tata McGraw-Hill, 2004.

[3] S. Willard, *General Topology*, Addison Wesley, New York, 1968.

[4] J. Dugundji, Topology, Allyn and Bacon, Boston, 1966.

[5] J. L. Kelley, *General Topology*, Van Nostrand Reinhold Co., New York, 1955.

[6] E. H. Spanier, *Algebraic Topology*, McGraw-Hill, 1966.

MTS 557	Algebraic Number Theory	4 Credits (48 hours)

Course Objectives

1. **Explore Algebraic Numbers**: Recap field extensions, and study algebraic and transcendental numbers, Liouville's Theorem, algebraic number fields, and properties of algebraic integers, including norms, traces, and quadratic/cyclotomic fields.
2. **Understand Factorization into Irreducibles**: Investigate factorization into irreducibles, non-unique factorization examples, prime factorization, Euclidean domains, and unique factorization consequences, including the Ramanujan-Nagell Theorem.
3. **Study Factorization of Ideals**: Learn about integral closure, Dedekind domains, fractional ideals, and unique factorization within Dedekind domains, including the norm of an ideal and its properties.
4. **Analyze Ramification and Ideal Class Groups**: Examine the ramification index, different of algebraic number fields, Dedekind's Theorem, and splitting of primes in quadratic fields, along with elementary results on ideal class groups.

Course Outcomes

1. Ability to work with field extensions, algebraic and transcendental numbers, and algebraic integers, and apply Liouville's Theorem and properties of quadratic and cyclotomic fields.
2. Skill in factoring into irreducibles, understanding unique and non-unique factorization, and applying concepts from Euclidean domains and the Ramanujan-Nagell Theorem.
3. Competence in analyzing integral closures, Dedekind domains, fractional ideals, and unique factorization in Dedekind domains.

4. Proficiency in determining the ramification index, different of number fields, and splitting of primes, and understanding ideal class groups and their properties.
5. Ability to solve problems related to ideal class groups, Diophantine equations, and apply theoretical results to practical problems in algebraic number theory.

Unit I - Algebraic Numbers (12 Hours)

Recapitulation of Field Extensions and properties, Definition and Examples of algebraic and transcendental numbers, Liouville's Theorem, Algebraic Number Fields, Conjugates and Discriminants, Algebraic Integers, Integral Bases, Norms and Traces, Rings of Integers,Quadratic Fields and Cyclotomic Fields.

Unit II - Factorization into Irreducibles (12 Hours)

Trivial Factorizations, Factorization into Irreducibles, Examples of Non-Unique Factorizationinto Irreducibles, Prime Factorization, Euclidean Domains, Euclidean Quadratic Fields,Consequences of Unique Factorization, The Ramanujan-Nagell Theorem.

Unit III - Factorization of Ideals (12 Hours)

Integral closure, Dedekind domains - Definition, Characterizations, Fractional ideals and unique factorization, Norm of an ideal-Definition and Properties.

Unit IV (12 Hours)

Ramification index and degree of a prime ideal, Different of an algebraic number field, Dedekind's Theorem. Splitting of primes in quadratic fields
The ideal Class Group: Elementary results, Definitions, Finiteness of the ideal class group, Diophantine equations, Supplementary problems

References

[1] Jody Esmonde and M. Ramamurthy, Problems in Algebraic Number Theory, 2nd Ed. Springer Verlag, 2004.

[2] I. N. Stewart and David Tall, Algebraic Number Theory and Fermat's Last Theorem, A. K. Peters Ltd., 2002.

[3] Saban Alaca and Kenneth S. Williams, Introductory Algebraic Number Theory, Cambridge University Press, 2004.

[4] Pierre Samuel, Algebraic Theory of Numbers, Dover Publications, 2008.

[5] Karlheinz Spindler, Abstract Algebra with Applications, Vol. II, Rings and Fields, Marcel Dekkar, Inc, 1994.

MTS 558	**Calculus of Variations and Integral Equations**	**4 Credits (48 hours)**

Course Objectives:

1. To introduce the fundamental concepts of variational problems and integral equations.

2. To provide an understanding of solving variational problems with fixed and moving boundaries.
3. To familiarize students with the application of integral equations in solving differential equations.
4. To develop the ability to analyze and solve complex mathematical problems using variational and integral techniques.

Course Outcomes:

1. Understand the foundational principles of the calculus of variations and its applications.
2. Solve variational problems with both fixed and moving boundaries.
3. Apply integral equations to convert and solve differential equations.
4. Analyze and solve Fredholm and Volterra integral equations using various methods.
5. Gain proficiency in the theoretical and practical aspects of integral equations and variational problems.

Contents

Unit I - Variational Problems with the Fixed Boundaries (12 Hours)

Introduction, problem of brachistochrone, problem of geodesics, isoperimetric problem, Variation and its properties, functions and functionals, Comparison between the notion of extrema of a function and a functional. Variational problems with the fixed boundaries, Euler's equation, the fundamental lemma of the calculus of variations, examples, Functionals in the form of integrals, special cases containing only some of the variables, examples, Functionals involving more than one dependent variables and their first derivatives, the system of Euler's equations, Functionals depending on the higher derivatives of the dependent variables, Euler-Poisson equation, examples, Functionals containing several independent variables, Ostrogradsky equation, examples.

Unit II - Variational Problems with Moving Boundaries, Sufficiency Conditions (12 Hours)

Moving boundary problems with more than one dependent variables, transversality condition in a more general case, examples, Extremals with corners, refraction of extremals, examples,One-sided variations, conditions for one sided variations. Field of extremals, central field of extremals, Jacobi's condition, The Weierstrass function, a weak extremum, a strong extremum, The Legendre condition, examples, Transforming the Euler equations to the canonical form, Variational problems involving conditional extremum, examples, constraints involving severalvariables and their derivatives, Isoperimetric problems, examples.

Unit III - Integral Equations (12 Hours)

Introduction, Definitions and basic examples, Classification, Conversion of Volterra Equation to ODE, Conversion of IVP and BVP to Integral Equation. Fredholm's Integral equations - Decomposition, direct computation, Successive approximation, Successive substitution methods for Fredholm Integral Equations.

Unit IV- Voltera Integral Equations and Fredholm's theory (12 Hours)

Voltera Integral Equations: A domain decomposition, series solution, successive approximation, successive substitution method for Volterra Integral Equations, Volterra Integral Equation of first kind, Integral Equations with separable Kernel.

Fredholm's theory: Hilbert-Schmidt Theorem: Fredholm's first, second and third theorem, Integral Equations with symmetric kernel, Eigen function expansion, Hilbert-Schmidt theorem.

References

[1] R. Courant and D. Hilbert, *Methods of Mathematical Physics*, Vol I, Interscience Press, 1953.

[2] L. E. Elsgolc, *Calculus of Variations*, Pergamon Press Ltd., 1962.

[3] R. Weinstock, *Calculus of Variations with Applications to Physics and Engineering*, Dover, 1974.

[4] D. Porter and D. S. G. Stirling, Integral Equations, *A practical treatment from spectral theory and applications*, Cambridge University Press, 1990.

[5] R. P. Kanwal, *Linear Integral Equations Theory and Practise*, Academic Press 1971.

[6] A. M. Wazwaz, *A first course in integral equations*, World Scientific Press, 1997.

[7] C. Cordumeanu, *Integral Equations and Applications*, Cambridge University Press, 1991.

MTS 559	Mathematical Statistics	4 Credits (48 hours)

Course Objectives

1. **Understand Probability Concepts**: Define probability, conditional probability, and independence; apply Bayes' theorem, and understand random variables, distributions, expectations, and moment generating functions.
2. **Study Distributions**: Analyze standard discrete and continuous distributions, including Bernoulli, Binomial, Uniform, Exponential, Normal, and their properties, and apply transformation techniques and sampling distributions.
3. **Explore Random Sequences and Statistical Inference**: Study random sequences, convergence, and laws of large numbers. Apply the central limit theorem and methods of point estimation, including sufficiency, unbiasedness, and maximum likelihood estimation.
4. **Master Hypothesis Testing**: Understand hypothesis testing fundamentals, apply Neyman-Pearson lemma, and use various tests such as the likelihood ratio test, t-test, Chi-square test, and nonparametric tests in applications.

Course Outcomes

1. Ability to define and apply probability concepts, conditional probability, and distributions, including their functions and inequalities.
2. Competence in analyzing and applying properties of standard discrete and continuous distributions and using transformation and sampling distributions effectively.
3. Skill in handling random sequences, applying laws of large numbers, and performing point estimation using methods like maximum likelihood estimation.
4. Ability to perform hypothesis testing using various tests, including Neyman-Pearson, likelihood ratio, t-test, Chi-square test, and nonparametric tests.
5. Proficiency in applying probability theory, statistical inference, and hypothesis testing techniques to real-world problems and data analysis.

Unit I - Probability, Conditional Probability and Moments (12 Hours)

Sample space, class of events; Classical and Axiomatic definitions of Probability, their consequences. Conditional Probability, Independence, Bayes' theorem and applications. Random Variables, Distributions Functions, Probability Mass functions, Probability Density functions. Expectations, Moment generating function, Probability generating function, Chebyshev's and Jensen's inequalities and applications.

Unit II – Distributions (12 Hours)

Standard discrete distribution and their properties - Bernoulli, Binomial, Geometric, Negative Binomial, Poisson distributions. Standard continuous distribution and their properties - Uniform, Exponential, Normal, Beta, Gamma distributions. Functions of random variables - transformation technique and applications, Sampling distributions - t, Chi-square, F and their properties.

Unit III - Random Sequences, Statistical Inference (12 Hours)

Sequences of random variables - Convergence in distribution and in probability, Chebyshev's, Weak law of large numbers. Central limit theorem and applications. Point estimation-sufficiency, unbiasedness, method of moments, maximum likelihood estimation.

Unit IV - Testing Hypothesis (12 Hours)

Testing of hypotheses - Basic concepts, Neyman-Person lemma, MP test. Likelihood ratio tests, *t*-test, Chi-square test and their applications. Nonparametric tests and their applications - Sign, Wilcoxon sign ranktest, Run test.

References

[1] Rohatgi V. K., *An introduction to probability theory and mathematical statistics*, Wiley Eastern ltd, 1985.

[2] Bhat B. R., *Modern Probability Theory*, an introductory text, Wiley eastern Ltd, 1981.

[3] Robert B Ash, *Probability and Mathematical Statistics*, Academic Press, Inc. NY, 1972.

[4] Hogg R.V. and Craig A. T., *Introduction to Mathematical Statistics*, 6th Ed., McMillan and Co., 2004.

[5] E. L. Lehmann and J. P. Romano, *Testing Statistical Hypothesis*, 3rd Ed., Springer, 2005.

[6] Freund, J.F., *Mathematical Statistics*, 8th Ed., Prentice Hall India, 2012.

MTS 560	**Computational Geometry**	**4 Credits (48 hours)**

Course Objectives:

1. To understand the fundamental concepts and computational techniques related to shape operators and surface geometry.
2. To study the intrinsic and extrinsic geometry of surfaces in Euclidean space and their global properties.
3. To explore the principles of Riemannian geometry and its applications in surface analysis.
4. To gain proficiency in computer-aided geometric design, including Bezier curves, spline curves, and interpolation methods.

Course Outcomes:

1. Comprehend and apply the concepts of shape operators and normal curvature in various computational scenarios.
2. Analyze the intrinsic and extrinsic properties of surfaces in E3E^3E3 and understand their fundamental equations and theorems.
3. Utilize the principles of Riemannian geometry, including covariant derivatives and geodesics, in surface construction and analysis.

4. Implement computer-aided geometric design techniques, including Bezier and spline curves, for practical geometric modeling.
5. Apply interpolation methods and smoothness conditions to create and refine geometric designs in a computational setting.

Contents

Unit I - Shape Operators:

The shape operator $M \subseteq R^3$, Normal curvature, Gaussian curvature, Computational Techniques, The implicit case. (8 Hours)

Unit II - Geometry of Surfaces in E^3:

The fundamental equations, Adapted frame field, Form computations, Some global theorems, Leibmann theorem, Isometries and local isometries, Intrinsic geometry of surfaces in R^3, Orthogonal coordinates, Congruence of surfaces. (10 Hours)

Unit III - Riemannian Geometry:

Geometric surfaces, Construction methods, Conformal change, Pull back, Coordinate description, Gaussian curvature, Theoremaegregium, Examples: flat torus, stereographic sphere, the stereographic plane, hyperbolic plane, the projective plane, tangent surfaces.

Covariant derivative: covariant derivative of R^2, parallel vector field, Geodesics, complete geometric surface, Liouville's formula.. (12 Hours)

Unit IV- Computer aided geometric design:

Bezier curves, de casterljau algorithm, properties of Bezier curves, Bloossom. Bernstein form of a Bezier curve, Derivative of Bezier curve, Subdivision, Bloossom and polar, Degree elevation, Variation diminishing property, Degree reduction, Non parametric curves, Cross plots, Different forms of a Bazier curve, Weierstrass approximation theorem, Formulas for Berstein polynomials. (10 Hours)

Unit V- Computer aided geometric design:

Interpolation by polynomial curves, Aitken's algorithm, spline curves in Bazier form, Smoothness conditions. C^1 and in C^2 continuity conditions, C^1 quadratic and C^2 cubic B-spline curves, parametrization, C^1 piecewise cubic interpolation, Cubic spline interpolation. (8 Hours)

References

[1] Barrett O' Neil, *Elementary differential geometry*, Academic Press, New York and London.

[2] G Farin, *Curves and Surfaces for Computer Aided geometric Design*, Academic Press.

[3] D. J. Struik, *Lectures on Classical Differential Geometry*, Addison Wesley Reading, Massachusetts, 1961.

[4] L. P. Eisenhart, *Riemannian Geometry, Princeton University Press*, Princetion, New Jersey, 1949.

[5] R. L. Bishop and S. J. Goldberg, *Tensor analysis on manifolds*, Macmillan co., 1968.

MTS 561	Cryptography	4 Credits (48 hours)

Course Objectives:

1. To provide a comprehensive understanding of classical and modern cryptographic techniques and their applications.
2. To develop the ability to analyze and implement various encryption and decryption algorithms.
3. To explore the mathematical foundations of public key cryptography and its practical implications.
4. To introduce advanced cryptographic concepts such as quantum cryptography, zero-knowledge proofs, and blockchain technology.

Course Outcomes:

Upon successful completion of this course, students will be able to:

1. Understand the history, importance, and basic concepts of cryptography, including encryption, decryption, and cryptanalysis.
2. Apply classical cryptographic methods such as Caesar cipher, Vigenère cipher, and one-time pad, and understand Shannon's theory of secrecy and perfect secrecy.
3. Analyze and implement modern symmetric key cryptographic systems, including block ciphers (DES, AES), stream ciphers, and cryptographic hash functions (MD5, SHA-1, SHA-2).
4. Understand and apply public key cryptographic techniques, including RSA algorithm, Diffie-Hellman key exchange, and Elliptic Curve Cryptography (ECC).
5. Explore and understand advanced topics in cryptography such as quantum cryptography, zero-knowledge proofs, homomorphic encryption, and blockchain technology, and keep abreast of current research trends and challenges in the field.

Contents

Unit I: Classical Cryptography and Basic Concepts (12 Hours)

Introduction to classical cryptographic systems, History and importance of cryptography, Basic terminology and concepts: encryption, decryption, and cryptanalysis, Classical ciphers: Caesar cipher, Vigenère cipher, and one-time pad, Principles of Shannon's theory of secrecy, Concept of perfect secrecy, Introduction to information theory in cryptography

Unit II: Modern Symmetric Key Cryptography (12 Hours)

Modern symmetric key cryptographic systems, Block ciphers: design principles and modes of operation, Data Encryption Standard (DES), Advanced Encryption Standard (AES), Cryptanalysis of block ciphers, Stream ciphers: design principles and applications, Cryptographic hash functions: MD5, SHA-1, SHA-2, Applications of hash functions: data integrity and authentication.

Unit III: Public Key Cryptography (12 Hours)

RSA algorithm: mathematical foundations, key generation, encryption, decryption, and digital signatures, Security aspects of RSA, Diffie-Hellman key exchange protocol, Elliptic Curve Cryptography (ECC): advantages and practical applications, Public Key Infrastructure (PKI) and certificate authorities.

Unit IV: Advanced Topics in Cryptography (12 Hours)

Quantum cryptography: principles and quantum key distribution (QKD), Potential impact of quantum cryptography on classical cryptographic systems, Zero-knowledge proofs: concepts and applications, Homomorphic encryption: principles and applications, Blockchain technology: cryptographic foundations and applications' Current research trends and challenges in cryptography.

References

[1] Katz, J., & Lindell, Y., "Introduction to Modern Cryptography," CRC Press, 2nd Edition, 2014.

[2] Stinson, D. R., "Cryptography: Theory and Practice," Chapman and Hall/CRC, 3rd Edition, 2005.

[3] Schneier, B., "Applied Cryptography: Protocols, Algorithms, and Source Code in C," Wiley, 2nd Edition, 1996.

[4] Menezes, A. J., van Oorschot, P. C., & Vanstone, S. A., "Handbook of Applied Cryptography," CRC Press, 1996.Neal Koblitz, *A course in Number Theory and Cryptography*, Springer Verlag, NewYork, 1987.

[5] Hans Delfs, Helmut Knebl*, Introduction to Cryptography*, Springer Verlag, 2002.

[6] William Stallings*, Cryptography and Network Security*, Prentice Hall of India, 2000.

[7] Alfred J. Menezes, Paul C. Van Oorschot, Scott A. Vanstone, *Handbook of Applied Cryptography*, CRC Press, 2000.

MTS 562	Finite Element Method with Applications	4 Credits (48 hours)

Course Objectives:

1. To introduce the fundamental principles and methods of weighted residual approximations and their application in solving differential equations.
2. To develop an understanding of finite element basis functions and their use in one-dimensional and two-dimensional problems.
3. To provide knowledge of finite element procedures, including formulation, element matrix calculation, and solution of linear equations.
4. To apply finite element methods to solve practical problems involving ordinary and partial differential equations, including applications in linear elasticity.

Course Outcomes:

Upon successful completion of this course, students will be able to:

1. Understand and apply weighted residual approximation methods such as point collocation, Galerkin, and least squares methods to solve differential equations.
2. Develop and use one-dimensional and two-dimensional basis functions, including Lagrange and serendipity family elements, for various geometrical shapes and coordinate transformations.
3. Formulate finite element methods for solving ordinary and partial differential equations, and perform the calculation of element matrices, assembly, and solution of linear equations.

4. Implement finite element solutions to practical problems, including one-dimensional ordinary differential equations, Laplace and Poisson equations in different domains, and applications in linear elasticity such as torsion of shafts with various cross-sectional shapes.

Contents

Unit I

Weighted Residual Approximations:- Point collocation, Galerkin and Least Squares method. Use of trial functions to the solution of differential equations. (12 Hours)

Unit II

Finite Elements:- One dimensional and two dimensional basis functions, Lagrange and serendipity family elements for quadrilaterals and triangular shapes. Isoparametric coordinate transformation. Area coordinates standard 2- squares and unit triangles in natural coordinates. (12 Hours)

Unit III

Finite Element Procedures:- Finite Element Formulations for the solutions of ordinary and partial differential equations: Calculation of element matrices, assembly and solution of linear equations. (12 Hours)

Unit IV

Finite Element solution of one dimensional ordinary differential equations, Laplace and Poisson equations over rectangular and nonrectangular and curved domains. Applications to some problems in linear elasticity: Torsion of shafts of a square, elliptic and triangular cross sections. (12 Hours)

References

[1] O.C. Zienkiewiez and K. Morgan, *Finite Elements and approximation*, John Wieley, 1983

[2] P.E. Lewis and J.P. Ward, *The Finite element method- Principles and applications*, Addison Weley, 1991

[3] L. J. Segerlind, *Applied finite element analysis* (2nd Edition), John Wiley, 1984

[4] O. C. Zienkiewicz and R. L. Taylor : *The finite element method*. Vol.1 Basic formulation and Linear problems, 4th Edition, New York, Mc.Graw-Hill, 1989.

[5] A.R. Mitchell and R. Wait, *Finite Element methods in Partial Differential Equations*, John Wiley, 1997.

[6] J.N. Reddy, *An introduction to finite element method*, New York, Mc.Graw Hill, 1984.

[7] D.W. Pepper and J.C. Heinrich: *The finite element method, Basic concepts and applications, Hemisphere,* Publishing Corporation, Washington, 1992.

[8] S.S. Rao, *The finite element method in Engineering*, 2nd Edition, Oxford, Pergamon Press, 1989.

[9] D. V. Hutton, *Fundamental of Finite Element Analysis*, (2004).

[10] E. G. Thomson*, Introduction to Finite Elements Method,* Theory Programming and applications, Wiley Student Edition, (2005).

MTS 563	Advanced Graph Theory	4 Credits (48 hours)

Unit I : Independent Sets and Matching

Introduction, Vertex- independent sets and Vertex coloring, Edge-independent sets, Matchings and Factors, Matchings in bipartite graphs, Perfect matchings and Tutte Matrix.

(12 Hours)

Unit II : Triangulated graphs

Introduction, Perfect graphs, Triangulated graphs, Interval graphs, Bipartite graph $B(G)$ of a graph G, Circular arc graphs. (12 Hours)

Unit III : Domination and Distances in Graphs

Domination numbers, Some elementary properties Bounds for domination number, Independent domination and irredundance. Center, Periphery of a Graph and properties.

(12 Hours)

Unit IV : Spectral properties of graphs

Introduction, The spectrum of a graph, Spectrum of the complete graph, spectrum of the cycle, Spectra of regular graphs, Spectra of complete bipartite graphs, The determinant of the adjacency matrix of a graph, Spectra of product graphs. (12 Hours)

References

[1] R. Balakrishnan and K. Ranganathan, *A textbook of Graph Theory*, Springer-Verlag, 2000.

[2] F. Buckley and F. Harary, *Distance in Graphs*, Addison-Wesley Publishing Company, 1990.

[3] Gary Chrtrand and Ping Zhang, *Introduction to Graph Theory*, Tata McGraw-Hill edition India, 2012.

[4] T. W. Haynes, S. T. Hedetniemi, P. J. Slater Fundamentals of Domination in graphs Maracel Dekker, INC. New York, 1998.

[5] Norman Biggs, *Algebraic Graph theory*, 2nd Ed. Cambridge Mathematical Library 1993.

[6] Douglass B. West, *Introduction to Graph Theory*, Prentice Hall of India, New Delhi, 1996.

[7] O. Ore, *Theory of Graphs*, American Mathematical Society, Providence, Rhode Island, 1967.

MTS 563	Fluid Dynamics	4 Credits (48 hours)

Course Objectives

1. To provide a comprehensive understanding of the fundamental concepts and principles of fluid dynamics.

2. To develop skills in analyzing and solving fluid flow problems using theoretical and practical approaches.
3. To introduce advanced topics in fluid dynamics, including boundary layer theory, compressible flow, and turbulence.
4. To equip students with the knowledge of numerical methods and computational techniques in fluid dynamics.

Course Outcomes

Upon completion of this course, students will be able to:

1. Understand and explain the basic concepts and principles of fluid mechanics and fluid properties.
2. Apply fundamental equations of fluid dynamics to analyze and solve fluid flow problems in various contexts.
3. Analyze fluid flow in pipes and channels using continuity, Euler's, and Bernoulli's equations.
4. Understand and apply boundary layer theory to analyze external flow problems around bodies.
5. Comprehend advanced topics such as compressible flow, shock waves, and turbulence, and apply numerical methods for fluid flow analysis.

Contents

Unit I: Introduction to Fluid Dynamics and Basic Concepts

Introduction to Fluid Mechanics: Definition of fluid mechanics, Classification of fluids, Continuum hypothesis. Fluid Properties: Density, Viscosity, Surface Tension, and Compressibility. Fluid Statics: Pressure Variation in a Fluid, Hydrostatic Pressure, Pascal's Law, and Manometry. Basic Fluid Dynamics: Continuity Equation, Euler's Equation of Motion, Bernoulli's Equation, and Applications.

(12 Hours)

Unit II: Fluid Flow Analysis

Kinematics of Fluid Flow: Types of Flow (Steady, Unsteady, Laminar, Turbulent), Flow Visualization, Streamlines, and Pathlines. Fundamental Equations: Navier-Stokes Equations, Derivation and Interpretation, Incompressible and Compressible Flows. Flow in Pipes and Channels: Flow Rate, Velocity Distribution, Darcy-Weisbach Equation, and Head Loss. Dimensional Analysis: Buckingham Pi Theorem, Non-Dimensional Numbers (Reynolds Number, Froude Number), and Application to Fluid Flow Problems.

(12 Hours)

Unit III: Boundary Layers and External Flow

Boundary Layer Theory: Development of Boundary Layers, Boundary Layer Thickness, and Displacement Thickness. Laminar and Turbulent Boundary Layers: Characteristics and Separation, Kinematic and Dynamic Boundary Layer. External Flow Analysis: Flow Around Bodies, Drag and Lift, Streamline Flow, and Flow Separation. Flow in External Fields: Application of Bernoulli's Principle to External Flows, Lift and Drag Coefficients, and Airfoil Theory.

(12 Hours)

Unit IV: Advanced Topics in Fluid Dynamics

Compressible Flow: Basic Concepts of Compressibility, Shock Waves, and Mach Number. Fluid Dynamics of Compressible Fluids: Isentropic Flow, Normal Shock Relations, and Oblique Shock Waves. Numerical Methods in Fluid Dynamics: Finite Difference Methods for Fluid Flow, Computational Fluid Dynamics (CFD) Basics, and Applications. Introduction to Turbulence: Characteristics of Turbulent Flow, Reynolds Averaged Navier-Stokes (RANS) Equations, and Turbulence Modeling.

(12 Hours)

References

[1] Batchelor, G.K., *An Introduction to Fluid Dynamics*, Cambridge University Press, 1st Edition, 1967.

[2] White, F.M., *Fluid Mechanics*, McGraw-Hill Education, 8th Edition, 2015.

[3] Frank, M., *Fundamentals of Fluid Mechanics*, Wiley, 2nd Edition, 2002.

[4] Chung, T.J., *Computational Fluid Dynamics*, Cambridge University Press, 1st Edition, 2010.

[5] Landau, L.D., and Lifshitz, E.M., *Fluid Mechanics*, Pergamon Press, 2nd Edition, 1987.

[6] Panton, R.L., *Incompressible Fluid Flow*, Wiley, 3rd Edition, 2005.

[7] Kundu, P.K., and Cohen, I.M., *Fluid Mechanics*, Academic Press, 5th Edition, 2015.

[8] Hirsch, C., *Numerical Computation of Internal and External Flows*, Butterworth-Heinemann, 1st Edition, 1990.

Semester wise distribution of credits for M.Sc. Mathematics Programme

SEM	Theory(HC^a)		Theory (SC^b)		Open Elective		Lab Credits (SC^b)	Project Credits (HC^a)	Total Credits
	No. of Courses	Credits	No. of Courses	Credits	No. of Courses	Credits			
I	3	4	2	4	-	-	2	-	22
II	3	4	2	4	1	3	2	-	22+3
III	3	4	2	4	1	3	2	-	22+3
IV	2	4	2	4	-	-	-	4	20
Total		44		32	-	6	6	4	86+6

HC^a - Hard core, SC^b - Soft core, Not included for CGPA

Total Hard Core Credits is 44+4=48 (55:81%) and total Soft Core Credits is 32+6=38 (44:19%).